Tracking Error and Active Portfolio Management*

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Persistent bear market conditions have led to a shift of focus in the tracking error literature. Until recently the portfolio allocation literature focused on tracking error minimization as a consequence of passive benchmark management under portfolio weights, transaction costs and short selling constraints. Abysmal benchmark performance shifted the literatureís focus towards active portfolio strategies that aim at beating the benchmark while keeping tracking error within acceptable bounds. We investigate an active (dynamic) portfolio allocation strategy that exploits the predictability in the conditional variance-covariance matrix of asset returns. To illustrate our procedure we use Jorionís (2002) tracking error frontier methodology. We apply our model to a representative portfolio of Australian stocks over the period January 1999 through November 2002.

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1. Introduction

Persistent bear market conditions on stock markets worldwide have generated a bull market in the tracking error literature. Practitioner-oriented journals (in particular the Journal of Portfolio Management and the Journal of Asset Management, see our references) recently devoted whole issues to implementation and performance measurement of tracking error investment strategies. More technical ñ mathematically-inclined ñ journals (e.g., the International Journal of Theoretical and Applied Finance) publish ever-faster optimization

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and risk measurement algorithms for increasingly realistic portfolio dimensions. Not coincidentally, this surge in academic interest ëtracksí the global stock market slump with fund manager performance coming under intense scrutiny from investors.

With absolute return performance as a first-order condition of investorís utility preferences, tracking the benchmark as closely as possible is normally sufficient during bull market conditions. When the benchmark fails to deliver, however, fund managers will have to prove relative return performance against the benchmark. The aim is then to persistently outperform the benchmark. Of course, such performance will only be feasible if the manager is prepared to accept active risk, and hence incur a risk penalty. Most investment funds accept that investors want to cap this penalty and therefore set a maximum portfolio tracking error accordingly. Thus, tracking error can either be the investment goal, or an investment constraint. This leads to the following two interpretations of index tracking:

A passive strategy that seeks to reproduce as closely as possible an index or benchmark portfolio by minimizing the tracking error of the replicating portfolio;

or,

an active strategy that seeks to outperform an index or benchmark portfolio while staying within certain risk boundaries defined by the benchmark.

What distinguishes these strategies is the composition of total risk exposure. Both active and passive strategies will incur incidental risk, while the active strategy will also incur intentional risk. Intentional risk may consist of stock specific risk (active stock selection) or systematic risk (active benchmark timing). Interestingly, some index fund managers claim that ex ante passive indexing generates persistent above average returns ñ i.e., an active portfolio outcome. Of course, what they really mean is that passive indexing often outperforms the average active strategy (which is more likely a reflection of the poor active outcomes). A standard measure used to trade off active performance against intentional risk is the information ratio (a.k.a. appraisal ratio), defined as the portfolioís active return ñ the alpha ñ divided by the portfolioís active risk. This standardized performance measure can be used to assess ex ante opportunity but it is more frequently used to assess ex post achievement. Ex ante opportunity is defined by the maximum possible *IR* given a set of forecast stock returns (and their forecast risk measures) and an inefficient benchmark[1]. A passive manager (minimizing tracking error) will have an ex ante *IR* close to zero. An active manager (maximizing excess returns) will have a much larger *IR*. The ex post achieved *IR* will depend on the realized excess return over the benchmark and as such depend on the Information Coefficient (*IC*). The managerís *IC* measures the correlation between forecast excess returns and realized excess returns. Whereas the ex ante *IR* will (by necessity) be strictly non-negative, the ex post *IR* can of course be negative. For a comprehensive discussion of these performance measures, we refer to Grinold and Kahn (1999). Active managers perceive that they need to frequently reallocate their portfolios either to ëcaptureí excess returns or to stay within a tracking error constraint. At times this may lead to a less than perfectly diversified portfolio, and will incur substantial transaction costs and assume high total risk. Of course, few passive index fund managers hold portfolios that exactly match the index, e.g., due to liquidity constraints. Just as active management incurs transaction costs, the passive manager then also has to rebalance the ëindexí portfolio to match the actual index returns as closely as possible. This suggests that there is a fairly close symmetry in the treatment of active and passive tracking error strategies. The key difference in the interpretation of passive and active ex post *IR* is that the best active manager will be characterized by a persistently large positive *IR*, whereas the best passive manager will have an ex post *IR* close to zero.[2]

The passive tracking practitioners have been well served by the academic literature. Rudd (1980), Chan and Lakonishok (1993) and Chan, Karceski, and Lakonishok (1999) are but a few of the many examples of this literature. The active tracking practitioners have not yet attracted similar attention[3]. To the best of our knowledge, Roll (1992) and Jorion (2002) are the first papers that comprehensively derive and interpret active portfolio allocation

[1] If the benchmark happens to be an efficient portfolio, the ex ante maximum possible *IR* will be zero! Of course, this is highly unlikely in practice. Typical benchmarks like the S&P500 or the ASX200 are commonly found well below the efficient frontier.

[2] However, the measure can be poorly defined. Consider the passive manager who actually holds the benchmark portfolio. For this manager, the *IR* will not be defined.

[3] Most standard investment textbooks still discuss tracking error in a passive portfolio management context, see e.g., Elton et al. (2003, p.677). Tracking error is then typically defined as the standard error of a regression of the passive portfolio returns on benchmark returns, see also Treynor and Black (1973). This regression measure is appropriate if the ìbetaî in the regression equals 1 (as it would for passive portfolios), but it will overstate tracking error when this is not the case (as it would for active portfolios). The same applies to the correlation measure suggested in Ammann and Zimmermann (2001). We therefore define tracking error as the square root of the second moment of the deviations between active portfolio returns and benchmark returns. Alternatively, one can define tracking error as the mean absolute deviation between active portfolio and benchmark returns, see e.g., Satchell and Hwang (2001). Both definitions can be used for ex ante tracking error (using forecast active and benchmark returns) as well as ex post tracking error (using realized active and benchmark returns).

solutions within a tracking error context. There are a few papers (e.g., Clarke et al., 2002) that investigate different active strategies with or without constraints on weights and/or risk. Unlike Roll and Jorion they do not analytically trace the trade-off between active risk-taking and expected excess returns.

In this paper we apply Jorioní s approach to active portfolio management within a tracking error constrained environment. We extend the methodology by taking a careful (and practical) approach to compute the input list. We investigate the impact of seriously inefficient benchmarks (which Jorion excludes) and the introduction of short selling constraints. We apply the methodology to the top-30 stocks of the Australian Stock Exchange during a three year sample period characterized by a strong bull market followed by a sharp and persistent bear market. We find, not surprisingly, that market conditions have a substantial impact on the active portfolio allocation and its ex post performance. This becomes even more apparent when we allow for short selling constraints.

The next section briefly describes our methodology, by reviewing the well-known portfolio optimization algebra and the lesser-known tracking error analytical solutions. We also describe how we operationalize the general portfolio allocation model. Section 3 summarizes the data from the Australian Stock Exchange and illustrates typical implementation issues that confront portfolio managers. Section 4 discusses the empirical results of our tracking error optimization. We conclude with lessons learnt from this exercise and possible venues for further research.

2. Methodology

Our methodology is based on Jorion (2002) to derive a constrained tracking error frontier. Define an observation, or estimation, period [t-j,t] from which we derive the input list. Based on this input list we first compute the ëglobalí efficient frontier without restrictions on risk or weights (except for the usual full investment constraint). We then investigate the reduction in investment opportunities when we introduce a tracking error constraint, followed by a short selling constraint. The introduction of a benchmark leads to a tracking error frontier, from which we derive the (conditionally) optimal active portfolio allocation. We then track this active portfolioís performance over a subsequent tracking period [t+1,t+k] and compute the realized tracking error. We dynamically update the active portfolio allocation at different frequencies (daily, weekly, monthly). Each time we update the investment opportunity set, we also locate the new ëex anteí position of the previous periodís active portfolio relative to the updated tracking error frontier.

Computation of the input list (arguably the most important stage of portfolio management, see Zenti and Pallotta, 2002) tends to be inconsistent in practice. Return forecasting is a strictly separate exercise from risk forecasting. Stock analysts provide the portfolio manager with forecast returns. These are typically point estimates without matching confidence intervals, i.e., stock analysts do not generate prediction intervals, see Blair (2002). A stock analystís information set typically comprises accounting information, economic information, management information, etc. It would be extremely complicated to combine the uncertainty surrounding each information variable into a single confidence interval for the stock return forecast. That is, the accuracy of the forecast is hard to define and measure. Econometric forecast models are commonly based on a much smaller information set of fairly homogeneous variables (like historical returns, dividends, growth rates). Even for these models it is still a challenge to find the joint confidence interval. In the absence of an analytical solution, simulation techniques or scenario analysis may be used to generate the uncertainty measure. The highest density forecast region proposed in Blasco and Santamaria (2001) or the bootstrapped prediction densities in Blair (2002) would be more promising candidates to solve this problem. In the absence of analyst forecasts (as in our application), the portfolio manager may apply some version of the beta pricing model along the lines of Rosenberg and Guy (1976), and Rosenberg (1985). We do not use fundamental information ñ as the portfolio manager would ñ but instead opt for the following simplification of the BARRA model to forecast betas:

$$\beta_{f,t} = (1-\phi)\beta_{a,t} + \phi\beta_h \qquad (1)$$

where β_a is the beta computed over the estimation period (effectively derived from the forecast variance-covariance matrix as discussed below) and β_h is the long-run beta used to ësmoothí the forecast,[4] with a smoothing parameter ϕ. Of course, there are different techniques to choose the smoothing parameter (e.g., Bayesian updating, Maximum Likelihood Estimation) and the historical long-run beta. We do not focus on the selection of ϕ or β_h, but do investigate the sensitivity of our results to different values of ϕ. We then use stock i's smoothed forecast beta, β_f, to generate its forecast return

[4] In our application, β_h is computed as an ëexpandingí average, i.e., at each optimisation period we compute the long-run beta over the full sample period up to that date (alternatively one could use the average beta measured over the year preceding the observation period).

$$E_t\left(R_{i,t+1}\right) = R_t^f + \beta_{f,t}^i\left[E_t\left(R_{B,t+1}\right) - R_t^f\right] \qquad (2)$$

where the forecast return on the benchmark is its average return over the estimation period. Of course, our choice of a single factor model is a further simplification. Grinold and Kahn (1999) discuss more appropriate multifactor generalizations (including the BARRA risk factor model). Chan, Karceski and Lakonishokís (1999) multifactor model ñ though restricted to forecast (co)variances ñ is potentially suitable to generate the complete input list, both risk and return forecasts. Given that we do not intend to maximize the ex post Information Ratio, we do not pursue those more elaborate models at this stage.

As suggested in Blair (2002) and discussed above, portfolio managers often apply forecast risk models in complete isolation from forecast return models. They might adopt a number of modeling specifications. Most of these are based on the premise that a stockís volatility changes over time (as does its beta). The ARCH model by Engle (1982) and its many offspring have dominated the academic portfolio literature for the past two decades. Much progress has been made in achieving ever better fitting specifications for univariate time series. Multivariate extensions ñ quite crucial for portfolio applications ñ have not witnessed similar advances, the main obstacle being the dimensionality problem. A completely unrestricted time-varying variance-covariance matrix is almost impossible to estimate for realistic portfolio dimensions (let alone achieving convergence in a realistic time frame)[5]. Pragmatic solutions are therefore needed and we opt for the approach advocated by RiskMetricsô (1995). Forecast volatility is an exponentially weighted moving average (EWMA) of past squared returns for stock i:

$$\sigma_{i,t}^2 = (1-\lambda)\sum_{j=0}^{\infty}\lambda^j R_{i,t-j}^2 \qquad (3)$$

where λ, the decay factor, depends on how fast the mean level of squared returns changes over time. The more persistent (autocorrelated) the squared returns will be, the closer λ should be to 1. For highly persistent time series of financial returns, we find values of λ between 0.9 and 1. Given the choice of decay factor, we then determine how many past observations should be used to compute forecast volatility. For a tolerance level of 0.01 (when we consider the observationís impact on forecast volatility to have sufficiently ëdecayedí), and λ=0.9, we need about 40 historical observations. As in RiskMetricsô, we apply (3) to each element of the variance-covariance matrix. Theoretically, with a universe of n stocks we should have as many as n+(n)x(n-1)/2 unique λís and matching estimation periods. In our application this implies 465 individual variance forecast processes. While still computationally feasible (and certainly faster than a multivariate GARCH estimation of the same dimension), for our purposes we impose a single λ on all variance-covariance elements. The penalty for this choice is reduced precision in the forecast variance-covariance matrix. It would be worthwhile to further investigate whether there is a ìdiversificationî effect in this additional forecast error across stocks, which would effectively reduce this penalty.

Which brings us to the next step, portfolio optimization. Rudd and Rosenberg (1980) comprehensively derive the investment allocation problem for a restricted portfolio universe (that is, for example, only the top 50 liquid stocks on a particular exchange) to match empirical practice. Jorionís (2002) derivation is similar in style and we follow his notation. First, the standard constrained portfolio variance minimization problem

$$\begin{aligned} &\min_{w} w_P'\Omega w_P \\ &s.t. \quad w_p' \times \iota = 1 \\ &\qquad\;\; w_P' \times E = \mu \end{aligned} \tag{4}$$

where E is a vector of forecast returns and Ω is the forecast variance-covariance matrix of returns and μ is a target portfolio return. Optimization over portfolio weights w_P leads to the well known hyperbola in mean ñ standard deviation space:

$$\begin{aligned} &\sigma_P^2 = \frac{1}{D}\left[\mu_P - \frac{B}{C}\right]^2 + \frac{1}{C} \\ &B = E'\Omega^{-1}\iota \\ &C = \iota'\Omega^{-1}\iota \\ &D = E'\Omega^{-1}E - \frac{B^2}{C} \end{aligned} \tag{5}$$

The weights of the efficient portfolios that fall on the hyperbola are readily obtained for different values of the target return constraint. Easy to apply, but of course, as soon as additional constraints are added on (e.g., a short selling constraint), we lose this analytical result and have to numerically optimize to find the feasible investment opportunity set. Now, if we slightly refocus the optimization problem to reflect a ìsearchî for portfolio value added (in excess of a benchmark portfolio value), we get

$$\begin{aligned} &\max_{a} \quad a_P'E \\ &s.t. \quad a_P' \times \iota = 0 \\ &\qquad\;\; a_P'\Omega a_P \le \Sigma \end{aligned} \tag{6}$$

[5] A recent exception is Timmermann and Blake (2002) but their portfolio dimensions are small.

where the active weights a_P (in deviation from the benchmark weights, w_B) have to add to zero to satisfy our original full investment constraint. This guarantees that total active portfolio weights, w_B+a_P, still add up to one. Active risk (in deviation from benchmark risk) may not exceed tracking error target variance Σ. Jorion derives the active portfolio solution if we set the tracking error constraint exactly equal to the target excess variance as

$$a_P = \pm\sqrt{\frac{\Sigma}{D}}\Omega^{-1}\left[E-\frac{B}{C}\iota\right] \tag{7}$$

where B, C and D are as defined in (4). As Jorion notes, these are solutions in excess return versus excess risk space. For ease of comparison with the standard portfolio setup, we would rather have a representation in the mean (total return) versus standard deviation (total risk) space:

$$\begin{aligned} \max_a \quad & a_P' E \\ s.t. \quad & a_P' \times \iota = 0 \\ & a_P' \Omega a_P \le \Sigma \\ & (w_B + a_P)' \Omega (w_B + a_P) = \sigma_P^2 \end{aligned} \tag{6í}$$

which upon maximization results in the following ellipsoidal solutions (μ_P, σ_P)

$$\begin{aligned} D(\sigma_P^2 - \sigma_B^2 - \Sigma)^2 + 4\left(\sigma_B^2 - \frac{1}{C}\right)(\mu_P - \mu_B)^2 - 4\left(\mu_B - \frac{B}{C}\right)(\mu_P - \mu_B)(\sigma_P^2 - \sigma_B^2 - \Sigma) \\ - 4\Sigma\left[D\left(\sigma_B^2 - \frac{1}{C}\right) - \left(\mu_B - \frac{B}{C}\right)^2\right] = 0 \end{aligned} \tag{8}$$

as long as the benchmark (μ_B, σ_B) lies within the efficient set. The ellipse is vertically centred around the benchmark expected return, but it is horizontally centred around the benchmark variance plus tracking error variance. The ellipseís principal axis is horizontal if benchmark expected return coincides with the expected return of the global minimum variance portfolio ($\mu_{MVP}=B/C$). If $\mu_B>\mu_{MVP}$ ñ typical in a bullish market ñ it will have a positive slope; if $\mu_B<\mu_{MVP}$ ñ in a bearish market ñ it will have a negative slope. Jorion provides an extensive discussion of displacements of the tracking error ellipse for changes in target tracking error variance. For our purposes we just mention two more relevant metrics:

$$\begin{aligned} \mu_{MAX} &= \mu_B + \sqrt{D\Sigma} \\ \sigma_{MAX}^2 &= \sigma_B^2 + \Sigma + 2\left(\mu_B - \frac{B}{C}\right)\sqrt{\left(\frac{\Sigma}{D}\right)} \end{aligned} \tag{9}$$

for the mean and variance of the maximum expected excess return portfolio. Note that the portfolio manager can only generate excess returns by assuming tracking error. Note also that the penalty for this active portfolio allocation not only increases with tracking error variance but also with a more efficient benchmark portfolio (an increasing gap between μ_B and *B/C*). The more efficient the benchmark, the harder it will be to beat its performance.

We test the Jorionís methodology on real data. Two empirical features stand out for our application. The benchmark is frequently so inefficient, that its expected return falls below the expected return of the global minimum variance portfolio (*B/C*). Also, the unconstrained weights take completely unrealistic values during bear market conditions with huge short selling implications. We therefore add a short selling constraint. This complicates matters, as Jorion suggests. The numerical optimization of (4) with such a constraint poses no particular problems. Unfortunately, we cannot simply ëdistortí the tracking error ellipse by taking the intersection of the tracking error ellipse and the constrained mean-variance efficient frontier. We have to perform an integrated numerical optimization of (6í) including the short selling constraint. The solution set then becomes considerably thinner. Ultimately we look for the maximum excess return active portfolio (a single point) that satisfies all constraints simultaneously. There is no analytic solution for this problem. Numerical optimization is, however, quite feasible.

3. Data Issues

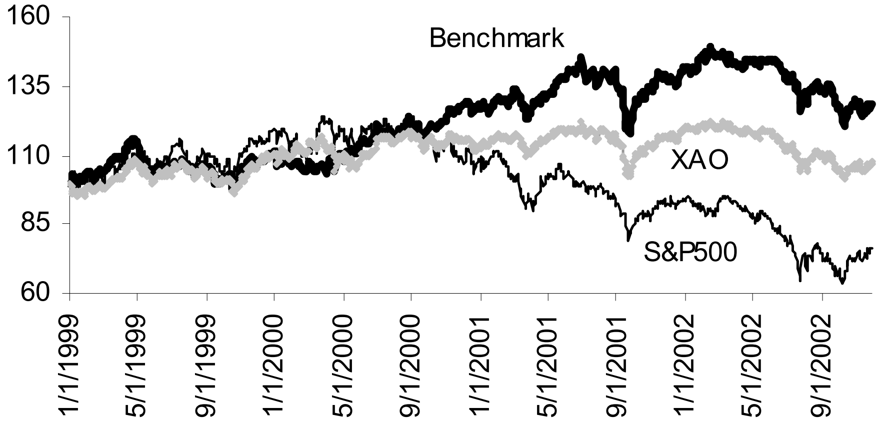
Our application considers a portfolio of 30 Australian stocks with a ëreducedí Australian All Ordinaries Index (XAO) as our benchmark. The stock price data and risk-free interest rates are obtained from Datastream and IRESS Market Technology. The top-30 stocks account for about 62% of the XAO index[6]. We standardize the top-30 weights to add up to 100% and generate a new top-30 benchmark accordingly. We choose a sample that covers the rather turbulent period from 4 January 1999 until 29 November 2002, a total of 991 trading days. What makes this sample particularly appealing is the strong bull market from the start of our sample until April 2000, followed by a sequence of collapses and persistent bear market conditions until the end of our sample.

Figure 1 indicates that our top-30 benchmark tracks the XAO index quite closely for the first year and a half of our sample. Mid 2000, however, the top-30 starts to diverge

[6] The top-10 stocks already account for over 40% of the XAO. For comparison, the top-10 stocks in the S&P500 account for about 25%. This dominance of a few stocks should make it theoretically easy to keep the tracking

substantially from the XAO index. The underperformance of the XAO index is easily explained by the under representation of technology stocks in the top-30, relative to the XAO. This is even more apparent when we include the S&P500 in the comparison. Whereas the S&P500 outperforms both top-30 and XAO indices until about June 2000, it has been much more exposed to the tech stocks collapse. A closer look at the key market drops in Figure 1 highlights this phenomenon.

Figure 1. All Ordinaries versus Top-30 Benchmark Index and S&P500



Note: Indices have been ëstandardizedí at 100 on the 1st of January 1999. The benchmark consists of the Top-30 stocks in the All Ordinaries.

Ranked by the size of the S&P500 collapse they are: the Nasdaq collapse on the 14th of April 2000, the 17th of September 2001 (following September 11), the 12th of March 2001, the 3rd of September 2002, the millennium bug on the 4th of January 2000 and the 19th of July 2002. The initial Nasdaq collapse is mirrored in the XAO collapse on the following trading day, but to a lesser extent for the top-30. Subsequent tech stock spillovers are less apparent ñ indicating the smaller presence of tech stocks in the Australian market. These US-led collapses have a moderate impact on the XAO index, but have virtually no impact on the top-30 index. Market wide collapses (like September 11, and the millennium bug), on the other hand, are similarly reflected in the XAO and the top-30 index. The lesser exposure to extremal price changes of the top-30 index does not imply less volatility. The standard deviation of returns is almost identical for both Australian indices (13% per annum), but both are substantially less volatile than S&P500 returns (a standard deviation of 22% per annum).

error within acceptable bounds. Our ex post tracking error results show that this is true with a short-selling

Figure 2 stresses the relevance of time-variation in the risk-free rate of return (the 90-day bill rate). It suggests that the bull market returns were somewhat tempered by a rising risk-free rate of return. Similarly bear market negative returns were offset by a decline in the risk-free rate of return (at least after October 2000). This offset lasts until April 2002, when the risk-free rate again starts to increase and risk-taking performance was doubly penalized (squeezing the excess returns).

Figure 2. Risk-Free Rate of Return

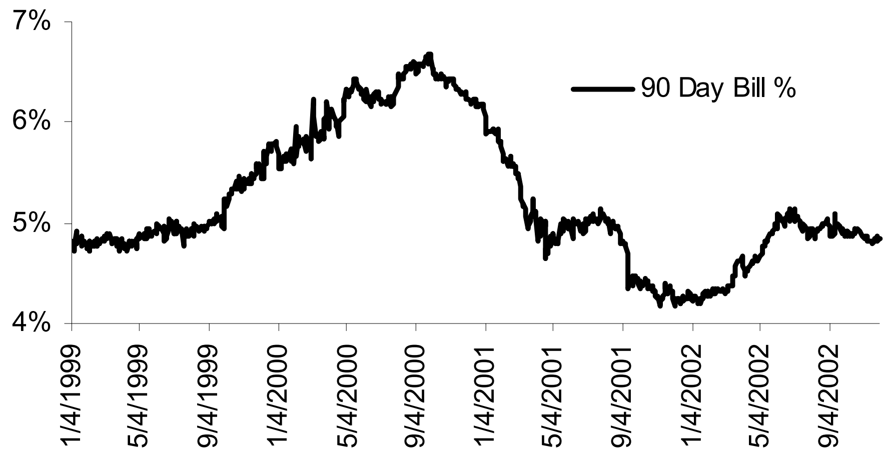


Table 1 summarizes the full sample input list. The top-30 stocks are listed by descending top-30 weight. Annualised mean returns and annualised standard deviations are given in columns 3 and 4, respectively. Volatility varies between a low of 15% (WFT) and a high of 53% (QBE), but is generally around 30% per annum. Risk compensation is rather meagre at a low of ñ20% (LLC) mean return to a high of 19% (WES) mean return. Our benchmark compares reasonably well with a mean return of 5% pa. (median return of 11% pa.) against a standard deviation of 13% pa. Taking into account that the risk-free rate of return varied between 4% and 7% pa., this does not suggest a generous excess return.

Table 1 also gives daily (not annualised) maximum return and minimum return in columns 5 and 6, respectively. The maxima vary between 4% and 25%, while the minima vary between ñ4% and ñ35%, truly extremal values. Not surprisingly, the empirical distributions of daily returns are hugely kurtotic (fat-tailed). This causes problems when we want to operationalise the input list for portfolio optimisation purposes. The non-normality of stock returns suggests that mean-variance optimization might not reflect investorsí utility tradeoff between risk and return.

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constraint, but is emphatically not true when short selling is possible.

Table 1. Descriptive Statistics for Top 30 ASX Stocks

Code	Top-30 weight	Annualized Average return	Annualized Standard deviation	Daily Maximum return	Daily Minimum return	Beta	Ljung-Box (p=12) Test
NAB	12.60	7.31	24.10	5.00	-13.87	1.18	25.87
CBA	9.37	4.06	20.80	3.97	-7.13	0.95	12.27
BHP	8.94	16.43	29.93	6.87	-7.62	1.21	16.72
TLS	7.75	-13.09	25.67	8.34	-9.86	0.91	20.49
WBC	6.96	6.75	21.12	4.87	-5.21	0.92	11.00
ANZ	6.81	14.42	21.99	4.71	-5.60	0.99	22.07
NCP	4.85	4.18	46.49	24.57	-14.89	1.95	9.18
AMP	4.14	-11.55	26.61	7.65	-8.03	1.03	17.16
RIO	4.14	14.88	30.02	7.18	-5.78	1.13	16.19
WOW	3.11	18.17	23.41	5.16	-8.34	0.53	28.93
WSF	2.38	13.38	29.62	20.92	-6.55	0.86	16.35
WES	2.31	19.03	26.78	10.58	-6.42	0.71	17.49
FGL	2.29	1.35	21.95	4.83	-7.06	0.49	36.79
WMC	2.28	13.41	34.32	16.32	-10.40	0.97	46.10
SGB	2.18	14.57	18.93	7.17	-5.02	0.53	6.92
WPL	2.05	12.55	27.02	7.57	-11.13	0.62	13.06
BIL	1.92	-19.41	33.86	9.54	-35.25	0.83	25.12
CML	1.90	-6.72	27.77	12.83	-18.18	0.61	10.62
WFT	1.67	-1.01	14.98	3.64	-4.38	0.38	20.28
CSR	1.54	11.46	27.07	9.16	-6.37	0.74	9.91
PBL	1.47	3.51	29.22	10.41	-10.83	0.82	21.62
GPT	1.31	-1.55	16.60	4.55	-3.98	0.42	19.58
CCL	1.20	-1.16	35.65	12.42	-10.80	0.69	13.43
MBL	1.20	10.35	26.65	7.11	-11.46	0.91	37.72
TEL	1.13	-13.72	29.44	9.63	-9.12	0.77	13.88
CSL	1.13	10.84	39.41	26.76	-12.01	0.76	22.29
LLC	1.05	-20.15	29.68	5.32	-17.11	0.72	36.05
QBE	0.98	4.67	52.83	41.85	-52.62	1.25	131.26
AGL	0.92	-3.61	22.54	4.42	-8.48	0.38	33.24
AXA	0.79	-4.04	29.61	7.89	-6.12	0.78	17.97
Index	**100.00**	**4.82**	**13.28**	**2.65**	**-5.14**	**1.00**	**11.83**

Note: Columns 2-6 are in percentages. The top-30 weights are ìfixedî as of November 2002 and are the standardized (they sum to 100%) XAO-weights on that date. We ignore any weight changes during our sample period. Betas are computed against the top-30 index. The Ljung-Box column gives the test statistic for serial correlation in the returns up to 12th order lag length, with a 95% critical value of 21.03.

Campbell et al. (2001) illustrate an alternative optimisation procedure that better captures the kurtotic (and perhaps skewed) nature of stock returns. As a matter for future research, we could incorporate an equivalent tracking error constraint in their optimisation procedure. As long as the empirical distributions are symmetric, we expect very little difference in terms of active weight selection.

Unlike stock analysts, we do not have the fundamentals (growth forecasts, accounting information, etc.) to value and then rank stocks by forecast return to generate buy/sell signals. Our information set is restricted to historical returns. For simplicity, consider the following example where our forecast return is a simple average of the past 5 trading daysí returns. If we happen to encounter a single extremal return in our information sample (say, 10.58%) and four zero returns, we generate an annualised forecast return of 535%. Clearly, the extremal return is non-representative for forecast purposes. Even for longer information samples (say a month, or 20 trading days) this inflation of annualised forecast returns based on extremal observations remains a problem. It reflects the fact that daily stock return distributions are not normally distributed, but are better characterized by some fat-tailed distribution (like, e.g., a Student-t). Unlike the normal distribution, a Student-t is not ìclosed under addition,î i.e., its properties (e.g., the variance) cannot be simply scaled to derive equivalents at a different sampling frequency (say from daily to annualised).

This problem highlights the difficulties that one encounters when optimising portfolios of individual stocks based on simplistic input list rules. It might explain why the academic literature prefers to build efficient portfolios from portfolios of individual stocks[7]. ëIndexingí clearly normalizes the empirical distributions (just consider the descriptive statistics for the top-30 index), which makes these portfolios much more suitable for portfolio optimisation purposes. This may be feasible for academic purposes, but it will not typically be a satisfactory solution for practical purposes. To somehow moderate the impact of extremal returns on our input list, we therefore choose to forecast returns based on a smoothed beta model as explained in the previous section.

Of course, the validity of using past average returns to forecast future returns (either as a simple average or as a market expectation) depends crucially on the stationarity of stock returns. The Ljung-Box portmanteau statistic (autocorrelation up to lag length 12) in column 8, Table 1, indicates that quite a few series display significant autocorrelation (95% critical

[7] The authors are well aware of more important reasons (like the errors-in-variables correction) for this choice.

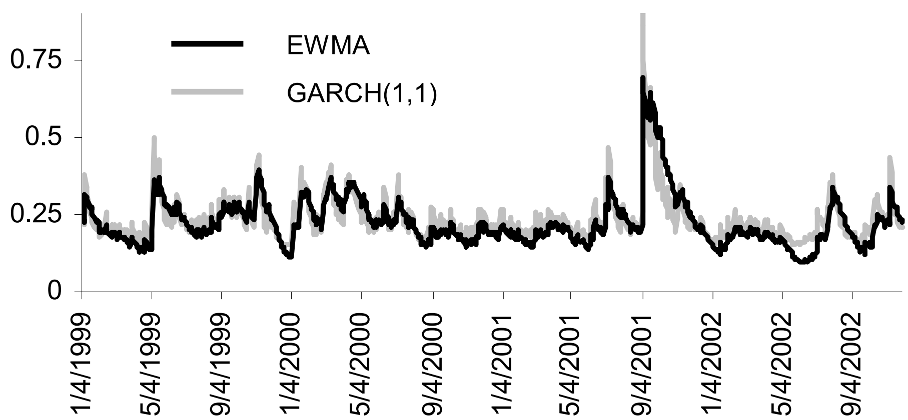
value χ^2_{12} = 21.03). As suggested by Lawton-Browne (2001), this may lead to a downward bias in ex ante tracking error. We investigate this below.

4. Empirical Results

To start our analysis, we first compute our input list. To update the vector of forecast returns, *E*, we use a simplified version of the BARRA beta pricing model encapsulated in equations (1) and (2). There is no real precedent for this procedure, and it obviously lends itself for future improvement. We choose ϕ=0.34 for our application, but also investigate the sensitivity of our results for different values of ϕ. This procedure generates fairly smoothly evolving forecast returns.

To update the forecast variance-covariance matrix, we use the RiskMetrics™ EWMA methodology. It is simple to understand, straightforward to implement, and generates GARCH-like variance processes. To illustrate this point, consider the GARCH(1,1) output in Figure 3 for NAB. The GARCH(1,1) parameters were estimated at α_1=0.17, α_2=0.74, while the average decay factor in the EWMA was estimated to be λ=0.91 (with an effective estimation sample length of 40 days). The EWMA process is somewhat smoother, but there seems little to separate the two processes ñ a similar point is made in RiskMetricsô (1995).

Figure 3. NAB Conditional Volatility ñ GARCH(1,1) versus EWMA

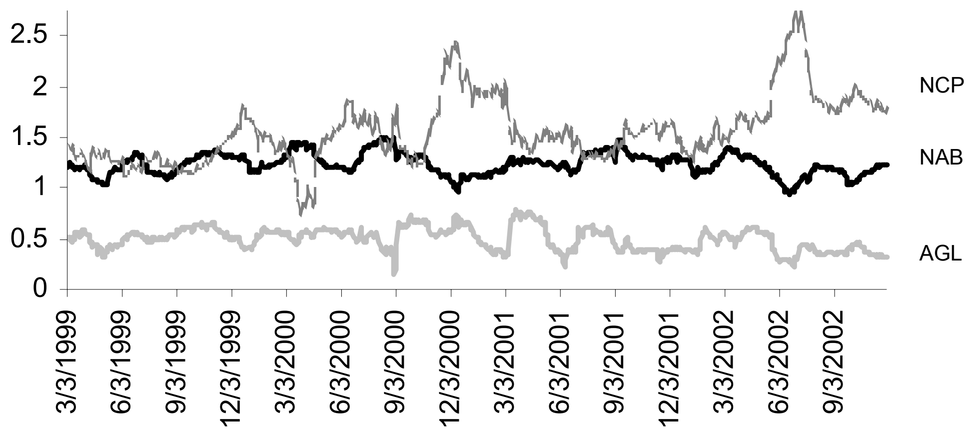


Note: Annualized standard deviations for NAB are based on fitting a GARCH(1,1) model: $\sigma^2_{it} = \alpha_0 + \alpha_1 R^2_{i,t-1} + \alpha_2 \sigma^2_{i,t-1}$, respectively an EWMA model: $\sigma^2_{it} = (1-\lambda)\sum_{j=0}^{\infty} \lambda^j R^2_{i,t-j}$.

Whereas it is relatively straightforward to estimate a univariate GARCH(1,1) process like the one above, the computational burden becomes excessive for a multivariate GARCH process involving 30 stocks. Scowcroft and Sefton (2001) investigate the performance of a number of time-varying risk matrix specifications ñ including the EWMA and GARCH models ñ and find that the tracking error predictions agreed reasonably well.

Figure 4 gives some insight in the dynamically updated input list for three stock components in the top-30 benchmark: NAB (the largest weight, 12.6% and a full-sample unconditional beta of 1.18), NCP (the highest full-sample unconditional beta, 1.95) and AGL (the lowest full-sample unconditional beta, 0.38). Figure 4 shows the time variation in their betas, according to equation (1).

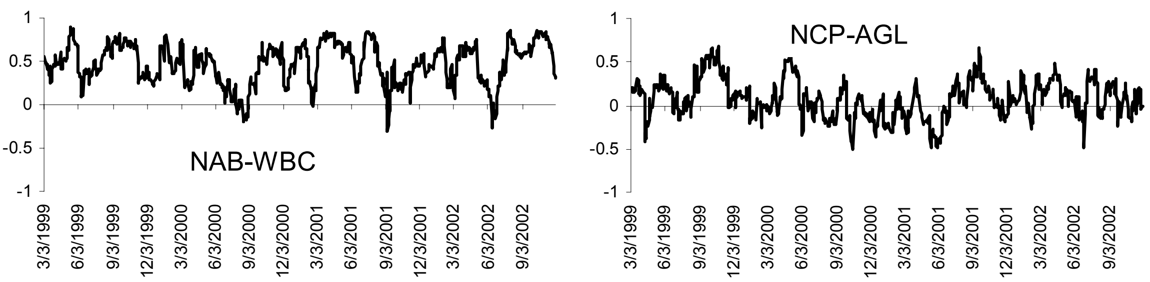
Figure 4. Time Variation in Betas



Note: Unconditional betas are respectively 1.18 (NAB), 1.95 (NCP), and 0.38 (AGL); Betas are measured against the top-30 benchmark index.

NAB has the more stable beta, whereas NCP and AGL (the stocks with more extreme ñ high and low ñ betas) have much more volatile intertemporal betas. This has obvious repercussions for the forecast returns according to equation (2), where the more volatile beta forecasts will generate more volatile stock return forecasts.

Figure 5. Time Variation in Correlation

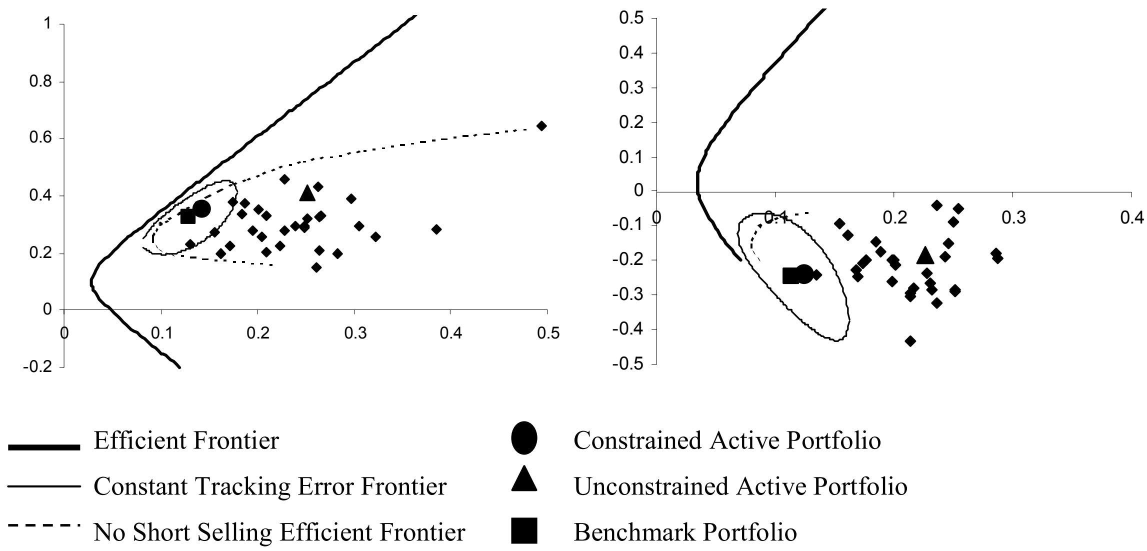


Note: NAB-WBC unconditional correlation =0.46; NCP-AGL unconditional correlation = 0.06;

Figure 5 illustrates the time-varying intra-sector correlation between NAB and WBC, two banking stocks in our universe; respectively between NCP and AGL for an inter-sector illustration. Intra- and inter-sector time-varying correlation are equally volatile, but obviously have a different mean. The intra-sector correlation displays switching behaviour with periods of high correlation alternating with periods of low correlation and not much in between. The inter-sector correlation does not share this feature.

Having completed the input list, we can proceed to compute the efficient frontier solving (4) with λ=0.91. Then we solve (6í) to obtain the active tracking error frontier with a tracking error target of 5%. Our top-30 benchmark is found to be seriously inefficient (throughout our sample period with a few exceptions when it is close to the global efficient frontier) which suggests active investment opportunities. Or, in ex ante Information Ratio terms, they offer a positive *IR* for active portfolio managers. To illustrate this, consider the following two representative optimization periods. Figures 6a and 6b are representative for a bull market episode, respectively a bear market episode. As expected in a bull (bear) market the active investment opportunity set ñ the ellipse ñ is upward (downward) sloping. The benchmark (the square symbol) typically has lower risk than the individual stocks (the diamond symbols) but of course, only average expected returns.

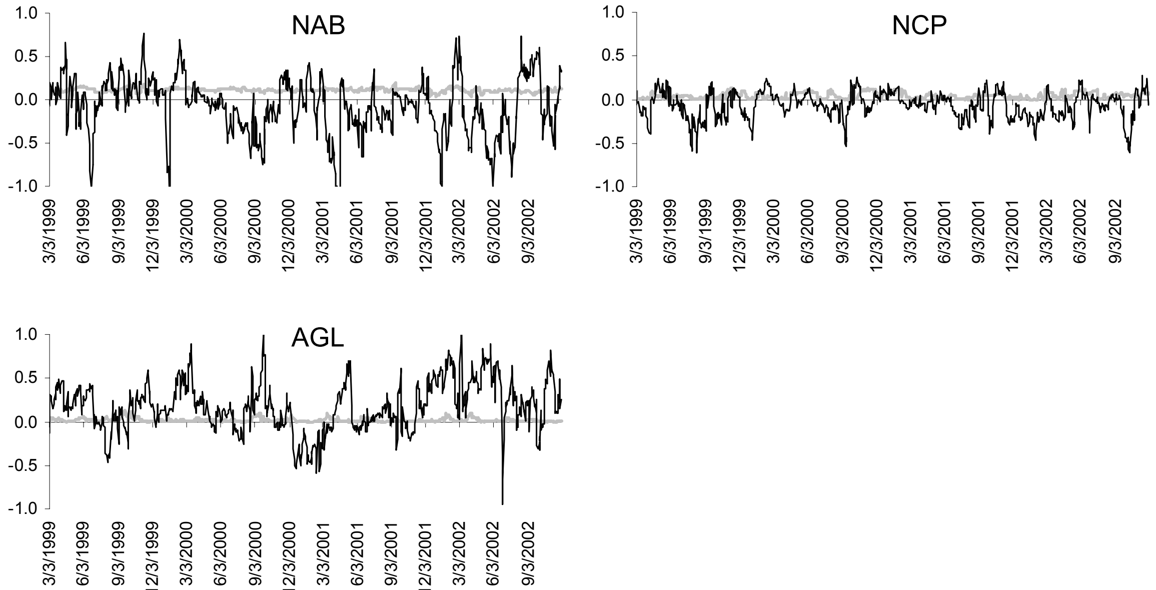
Figure 6. Bulls and Bears



The location of the active portfolios chosen in the previous period optimisation is also indicated in both graphs. Without rebalancing, the unconstrained active portfolio (circle symbol outside ellipse) needs no longer be on the updated ellipse. As it turns out, the constrained active portfolio (circle symbol inside ellipse) is always inside the updated ellipse, but the unconstrained active portfolio is almost without exception outside the updated tracking error frontier. That does not necessarily imply a violation of the tracking error constraint over that period, but does imply an ex ante tracking error violation for the subsequent period.

We summarize our dynamic optimisation exercise in Figure 7, which gives the active portfolio weights for three stocks (NAB, NCP, AGL) in the (un)constrained active portfolios. Perhaps surprisingly the unconstrained weights ñ the black lines ñ are as volatile (in both positive and negative direction) during the bull market as they are during the bear market. There is limited evidence of short-lived persistence in the active weights, suggesting that portfolios need to be rebalanced fairly frequently (at least monthly). The short selling constrained weights ñ the grey lines ñ are obviously much smoother which implies substantially less rebalancing costs.

Figure 7. Active portfolio weights for NAB, NCP and AGL

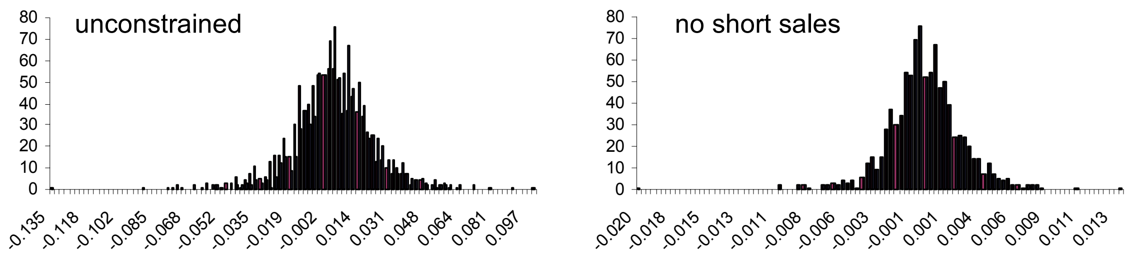


Note: Weights are given as fractions. Weights in the top-30 benchmark are respectively 0.126 (NAB), 0.049 (NCP), and 0.09 (AGL). Grey lines give no short selling weights; Black lines give unconstrained weights.

A comparison across stocks is interesting. The stock with the highest top-30 benchmark weight and a moderate beta (NAB), switches most frequently from positive to negative active positions. The stock with the lowest weight in the top-30 and the lowest beta (AGL), has predominantly positive active weights (although with substantial volatility). The stock with the highest beta (NCP) ñ not surprisingly the least ëAustralianí of the 30 stocks ñ has the most stable active weights (although they switch frequently from positive to negative and vice versa without much persistence).

Since we rebalance every time period, our ex ante tracking error is always exactly equal to the target. This is clearly not the case ex post. The problem now is how to measure the ex post tracking error. We choose to measure ex post performance using a quadratic tracking error measure,[8] that is the squared deviation of portfolio returns from benchmark returns. We first measure the ex post difference between portfolio return and benchmark return, i.e., the active return. This active/excess return performance is illustrated in Figure 8. A comparison suggests that the short selling constrained active portfolio only rarely ëwanders awayí from the benchmark (Figures 6a and 6b are fairly typical of this phenomenon). In fact, the constrained portfolio seems to ì hugî the benchmark over the tracking period.

Figure 8. Ex Post Active Returns



Note: the histograms have frequency count on the vertical axis and daily (fractional) excess returns on the horizontal axis.

The standard deviation of short selling constrained excess returns is almost exactly 5% per annum. Not entirely surprising given the visual evidence in Figure 8, the unconstrained standard deviation is 20% per annum. A nice illustration indeed of the theoretical result of the downward bias in ex ante tracking error as a forecast of ex post tracking error, Satchell and Hwang (2001). Further empirical evidence for this bias is given in Zenti and Pallotta (2002).

Among the possible reasons for this bias, Lawton-Browne (2001) and Scowcroft and Sefton (2001) suggest autocorrelation in returns and volatility clustering. We therefore investigate the impact of both violations of the mean-variance optimization assumptions. We also look at the sensitivity of our results to variations in tracking duration, and tracking error target. The results of these exercises are given in Tables 2a, 2b, and 2c.

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[8] Rudolf et al. (1999) suggest that a mean absolute deviation measure would be preferable since it matches fund managersí compensation schedules. We agree that this is worth investigating.

Table 2a. Ex Post Tracking Error ñ Varying tracking duration

Tracking Duration (days)	Constrained Tracking Error	Unconstrained Tracking Error
1	5.27%	19.67%
2	5.27%	19.93%
3	5.02%	19.68%
4	5.23%	19.77%
5	4.93%	20.54%
10	4.87%	19.76%
20	5.11%	19.85%
40	5.04%	19.79%
60	4.77%	18.20%
125	5.28%	22.25%

Note: tracking error target, $\sqrt{\Sigma}$=0.05; volatility persistence parameter λ=0.91.

Table 2b. Ex Post Tracking Error ñ Varying tracking error target

Tracking Error Target (st.dev)	Constrained Tracking Error	Unconstrained Tracking Error
1%	1.08%	3.97%
2.5%	2.59%	9.93%
5%	5.11%	19.85%
10%	7.57%	39.70%
20%	12.52%	79.41%

Note: tracking duration = 20 days; volatility persistence parameter λ=0.91.

Table 2c. Ex Post Tracking Error ñ Varying volatility decay factor

EWMA decay Factor (λ)	Constrained Tracking Error	Unconstrained Tracking Error
0.85	4.66%	50.84%
0.87	4.84%	36.40%
0.90	4.67%	15.25%
0.91	5.11%	19.85%
0.95	4.69%	12.62%
0.97	4.32%	11.66%
0.99	4.67%	15.25%

Note: tracking duration = 20 days; tracking error target, $\sqrt{\Sigma}$=0.05.

Table 2a gives ex post tracking errors (as a standard deviation of active returns) for a range of tracking durations (k=1,Ö ,125 days). Apparently, the duration of tracking does not materially affect the realized tracking error. Perhaps surprisingly, there is very little variation in ex post tracking error when rebalancing of the active portfolio occurs more or less frequently. Short selling constrained active portfolio management generates tracking errors that stay within the target tracking error. Unconstrained active portfolio management generates tracking errors well in excess of the target.

Table 2b gives ex post tracking error for a range of tracking error targets ($\sqrt{\Sigma}$=1%, Ö ,20%). We observe that the ex post short selling constrained tracking error marginally exceeds (is substantially less than) the ex ante tracking error for targets below (above) 5% per annum. Ex post unconstrained tracking error, however, generally exceeds the ex ante tracking error at an increasing rate with increasing target tracking error.

Table 2c suggests the source of the bias in ex ante tracking error expectations. It gives ex post tracking errors for a range of EWMA decay factors (λ=0.85, Ö ,0.99). For this (admittedly limited in scope) experiment, we find that at λ=0.97, the bias is minimized. We suspect that individual optimisation of the λ parameter for each and every element in the variance-covariance matrix would allow an even better outcome.

We also experimented with the serial correlation in stock returns by varying the smoothing parameter ϕ in (1). This did not affect the bias in ex ante tracking error. It does, of course, affect the location of the excess return distribution, but not the scale!

Bringing the two results (time-variation in weights and ex post performance) together suggests that the typical short selling constraint acts as a safeguard on ex post tracking error while simultaneously reducing the cost of rebalancing. A short selling constraint effectively turns active portfolio management into something very close to passive portfolio management. So why do we not observe the matching reduction in active returns? Simply because we made no real attempt to actively forecast stock returns. In fact, we have taken a rather ëpassiveí approach when generating the input list[9]. We can imagine that stock analysts (or sophisticated econometriciansí models for that matter) are able to manipulate these active performance distributions to their benefit and hence change the location of the active return distributions in Figure 8 accordingly.

5. Conclusions

Although straightforward in content, proper implementation of portfolio optimization theory can be notoriously complicated. Choices have to be made with regard to the input list, constraints, estimation procedure, and implied actions. This paper gives a flavour of some of the many issues that have to be dealt with by portfolio managers. The main advantage of our

[9] We use a mechanistic rule ñ as in equation (2) ñ to generate expected returns. Our choice of the smoothing parameter ϕ is not based on proper model selection criteria, since this was not the main focus of our paper.

procedure is its transparent nature which considerably facilitates communication with an ever more sophisticated clientele.

Jorionís (2002) main contribution is the visualization of the ìactive investment opportunity space.î We illustrate that the introduction of a short selling constraint eliminates most of this opportunity set ñ partly driven by persistent bear market conditions during our sample period. However, as an investment advice tool, successive introduction of investment constraints clearly identifies the location of (and the reduction in) the relevant investment opportunity set. The methodology also highlights the tradeoff between risk penalty and excess return gain when violating the investment constraints. From this perspective, Jorionís methodology is an invaluable contribution to the practical investment literature.

We find (as do many others, see e.g., Plaxco and Arnott, 2002) that frequent rebalancing is an absolute necessity to keep some control over total risk (though not necessarily tracking error risk) when actively managing portfolios. Larsen and Resnick (2001) consider a range of optimization and holding periods, but do not consider transaction cost constraints. Clearly, the costs of rebalancing have to be offset against the gains in risk control, but it seems to us that certain (threshold) levels of risk will simply be unacceptable. The issue, of course, is how to optimally rebalance so as to minimize the control costs.

Not surprisingly, we also find that ex ante tracking error expectations do not match ex post realizations, see also Rohweder (1998). Satchell and Hwang (2001) show that we can reasonably expect a worse ex post tracking error outcome due to the stochastic nature of portfolio weights. They report that this upward bias is not restricted to active portfolios, but can also be found in passive portfolios where the weights are not stochastic (due to rebalancing). A similar upward bias in ex post tracking error (but due to a different source) is caused by the apparent serial correlation, not just in the underlying stock returns, but also in the excess returns and squared excess returns. Frino and Gallagher (2001), e.g., find evidence of seasonality in tracking error (partly driven by seasonality in dividend payments on the benchmark). Pope and Yadavís (1994) results indicate that this will lead to a biased ex ante estimate of tracking error.

Where to from here? There is plenty of scope to improve the selection of optimal observation period and forecast period duration. Ultimately, this is an empirical matter. We hinted at the possibility (and Table 2b underlines its importance) to individualize the stochastic processes for every element of the variance-covariance matrix. A trade-off will then have to be made between the improvement in goodness-of-fit and the increased computational burden of such an exercise. Despite this, we argue that there is no urgent need to resort to computationally burdensome multivariate GARCH specifications.

Another constraint worth investigating is a cap on the number of stocks in the managed portfolio or the minimum number of stocks in an active portfolio. Jansen and van Dijk (2002) illustrate the small portfolio constraint for a passive tracking portfolio. Ammann and Zimmermann (2001) investigate admissible active weight ranges, which would guarantee a limit on individual stock weights. Alternatively, we could investigate a cap on the number of stocks in which the portfolio manager takes active positions, while taking neutral positions in the remaining benchmark component stocks. Yet another approach could be a factor-neutrality constraint as in Clarke et al. (2002), which would fit typical style-type portfolio constraints. It is quite possible that some of these constraints are internally conflicting. The long-only constraint, e.g., tends to favour small capitalization active stock weights, which would obviously clash with a large capitalization style constraint.

Another issue is the composition of the benchmark. Fund managers can frequently choose their benchmarks (within reasonable boundaries, i.e., among a peer group). Small cap fund managers would typically choose a representative small cap benchmark, like the ASX Small Ords. As shown in Larsen and Resnick (1998), the market capitalization of component stocks in the benchmark index has a non-trivial impact on the tracking performance of enhanced benchmark portfolios. Though their analysis quantifies the impact, they are not explicit on the source. It could be a liquidity constraint, or perhaps the (related) excessive non-normality of the returns of these stocks.

A few papers have recently focused on tracking error measurement that better reflects the incentive structure of the portfolio manager, see e.g., Kritzman (1987) and Rudolf et al. (1999). Roll (1992) suggests that diversification of an investorís portfolio across managers reduces the impact of excessively risky active portfolios. Jorion (2002) shows that this is not a satisfactory solution and instead favours additional constraints on total risk to better control for the free option provided to portfolio managers who are only constrained by active risk (or tracking error). A closer look at investorsí utility functions and better integration of these utility functions with constrained portfolio optimizations seems a worthwhile extension of this paper.

Perhaps most importantly, there needs to be more research towards proper integration of return and risk forecasts. Though it seems obvious that stock analysts do make risk assessments when computing forecast returns, it is much less obvious to extract and combine these risk assessments into a justifiable risk measure.

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