Weak Predictor-Corrector Schemes for Jump Diffusions in Finance

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Overview of the Presentation

- **jump-diffusion SDEs in finance**
- **strong vs weak numerical schemes**
- **jump-adapted weak numerical schemes**
- **numerical results**

Key Messages

- **task**
  - strong schemes $\Longrightarrow$ pathwise approximations
  - weak schemes $\Longrightarrow$ probability approximations

- **schemes**
  - explicit schemes $\Longrightarrow$ fast
  - implicit schemes $\Longrightarrow$ numerically stable
  - predictor-corrector $\Longrightarrow$ fast $+$ numerically stable

Problem Setting

- **multidimensional SDE with jumps**:

$$dX_t = a(t, X_t)dt + b(t, X_t)dW_t + c(t, X_{t-})\, dJ_t$$

- **jump-diffusion Merton model**

$$dX_t = X_{t-}\left(\mu dt + \sigma dW_t + \kappa dJ_t\right),$$

- **LIBOR market model with jumps** $\Rightarrow$ multi-dimensional non-linear drift

Numerical Approximations

- time discretization

$$t_n = n\Delta$$

- discrete time approximation

$$Y_n, \qquad n\{0, 1, \ldots \frac{T}{\Delta}\}$$

- strong schemes $\Longrightarrow$ pathwise approximations

- weak schemes $\Longrightarrow$ probability approximations

Strong Convergence

- **Applications:** scenario analysis, filtering and hedge simulation

- **Strong Convergence:**

$$\lim_{\Delta \to 0} E(|X_T - Y_N^\Delta|^2) = 0$$

- **Order:** $Y^\Delta \longrightarrow X$ with **strong order** γ if

$$\varepsilon_s(\Delta) = \sqrt{E(|X_T - Y_N^\Delta|^2)} \leq K\Delta^\gamma$$

Weak Convergence

- **Applications:** derivative pricing and evaluation of risk measures
- **Weak Convergence:**

$$\lim_{\Delta \to 0} |E(g(X_T)) - E(g(Y_N^\Delta))| = 0$$

examples: $g(x) = x^q, \quad g(x) = (x - K)^+, \quad g(x) = I_{\{x \leq c\}}$

- **Order:** $Y^\Delta \longrightarrow X$ with **weak order** β if

$$\varepsilon_w(\Delta) = |E(g(X_T)) - E(g(Y_N^\Delta))| \leq K\Delta^\beta$$

Example on a Diffusion

- **geometric Brownian motion**

$$dX_t = r\,X_t dt + \sigma\,X_t dW_t$$

- **Euler scheme** $(\gamma = 0.5,\ \beta = 1)$

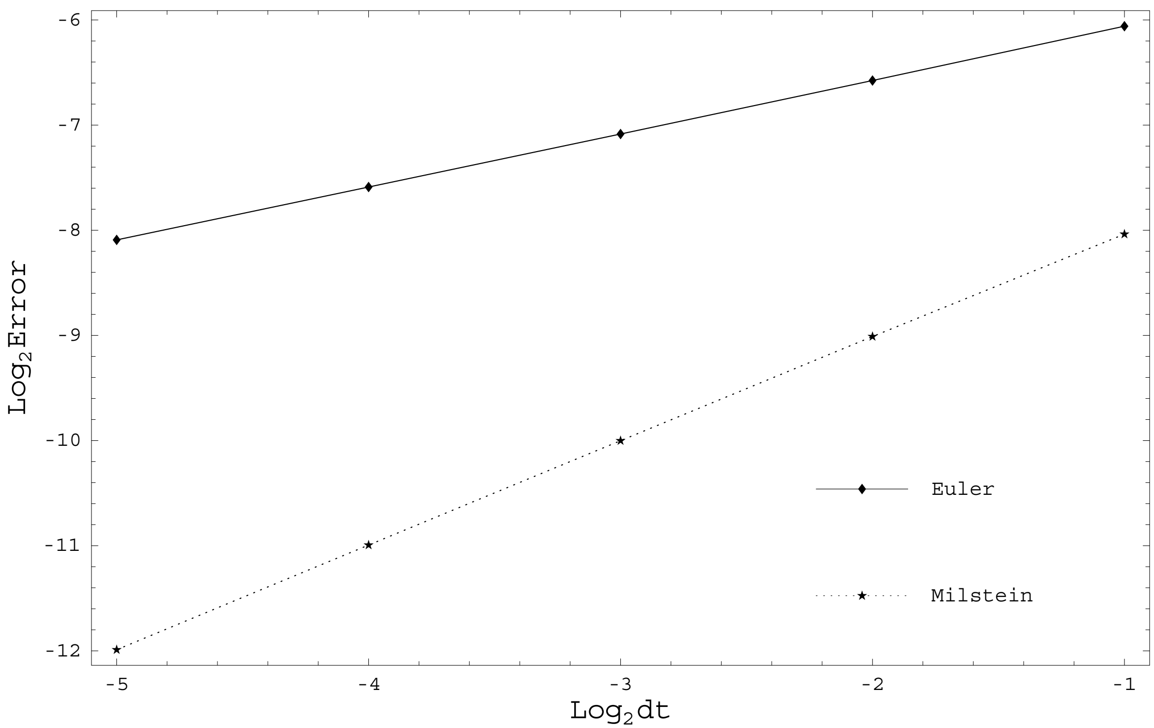
$$Y_{n+1} = Y_n + r\,Y_n \Delta + \sigma\,Y_n \Delta W_n$$

- **Milstein scheme** $(\gamma = 1,\ \beta = 1)$

$$Y_{n+1} = Y_n + r\,Y_n \Delta + \sigma\,Y_n \Delta W_n + \frac{\sigma^2}{2} Y_n \big((\Delta W_n)^2 - \Delta\big)$$

Strong Error

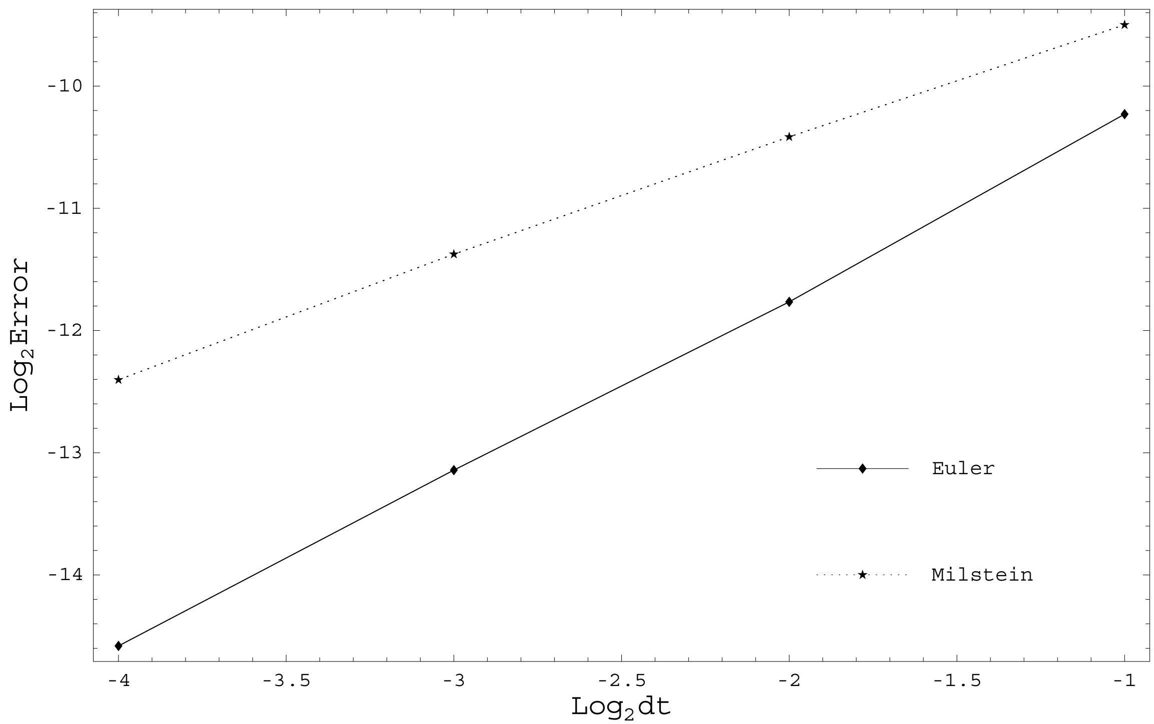
geometric brownian motion : $r = 0.05$, $\sigma = 0.2$, $X_0 = 1$, $T = 0.5$



Log-log plot of the strong error.

Weak Error

call payoff : $r = 0.05$, $\sigma = 0.2$, $X_0 = 1$, $T = 0.5$, $\mathrm{strike} = 1$



Log-log plot of the weak error.

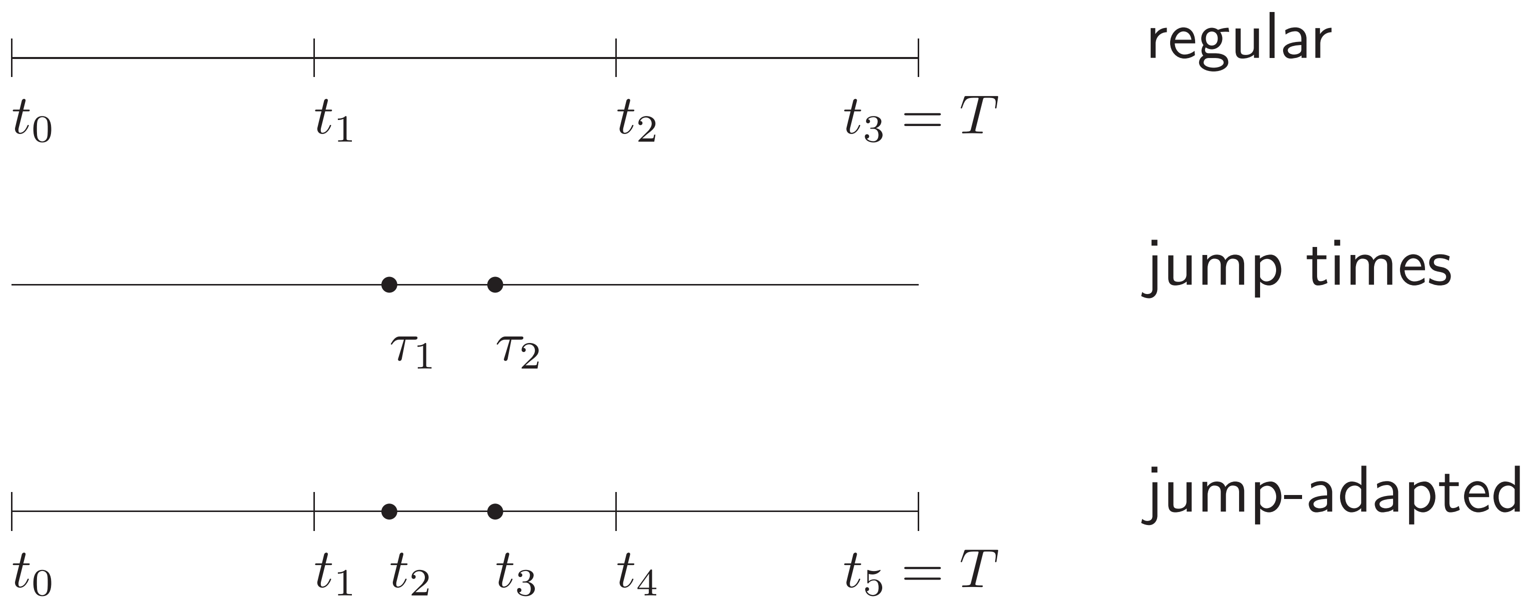
Literature of Predictor-Corrector Schemes

- Kloeden & Platen (1992) $\Rightarrow$ predictor-corrector for diffusions
- Platen (1995) $\Rightarrow$ other predictor-corrector for diffusions
- Hunter, Jackël & Joshi (2001), Joshi & Stacey (2006) $\Rightarrow$ predictor-corrector for LIBOR market models
- our contribution $\Rightarrow$ predictor-corrector for SDEs with jumps

Jump-Adapted Weak Approximations

jump-adapted time discretization

jump times included in time discretization



Jump-Adapted Weak Approximations

- **SDE**

$$dX_t = a(X_t)dt + b(X_t)dW_t + c(X_{t-})\, dJ_t$$

- **jump-adapted Euler scheme**

$$Y_{t_{n+1}-} = Y_{t_n} + a(Y_{t_n})\Delta_{t_n} + b(Y_{t_n})\Delta W_{t_n}$$

and

$$Y_{t_{n+1}} = Y_{t_{n+1}-} + c(Y_{t_{n+1}-})\big(J(t_{n+1}) - J(t_{n+1}-)\big)$$

- $\beta = 1$

Jump-Adapted order 1.0 weak Predictor-Corrector

- predictor-corrector $\Rightarrow$ numerical stability and efficiency
- **corrector ("implicit")**

$$Y_{t_{n+1}-} = Y_{t_n} + \frac{1}{2}\{a(\bar{Y}_{t_{n+1}-}) + a(Y_{t_n})\}\Delta + b(Y_{t_n})\Delta W_{t_n}$$

- **predictor (explicit)**

$$\bar{Y}_{t_{n+1}-} = Y_{t_n} + a(Y_{t_n})\Delta_{t_n} + b(Y_{t_n})\Delta W_{t_n}$$

- $\beta = 1$

Jump-Adapted order 2.0 weak Predictor-Corrector

- **corrector** $\Longrightarrow$ second order weak implicit scheme

- **predictor** $\Longrightarrow$second order weak explicit scheme

- $\beta = 2$

Numerical Results

- **jump-diffusion Merton model**
  - call payoff

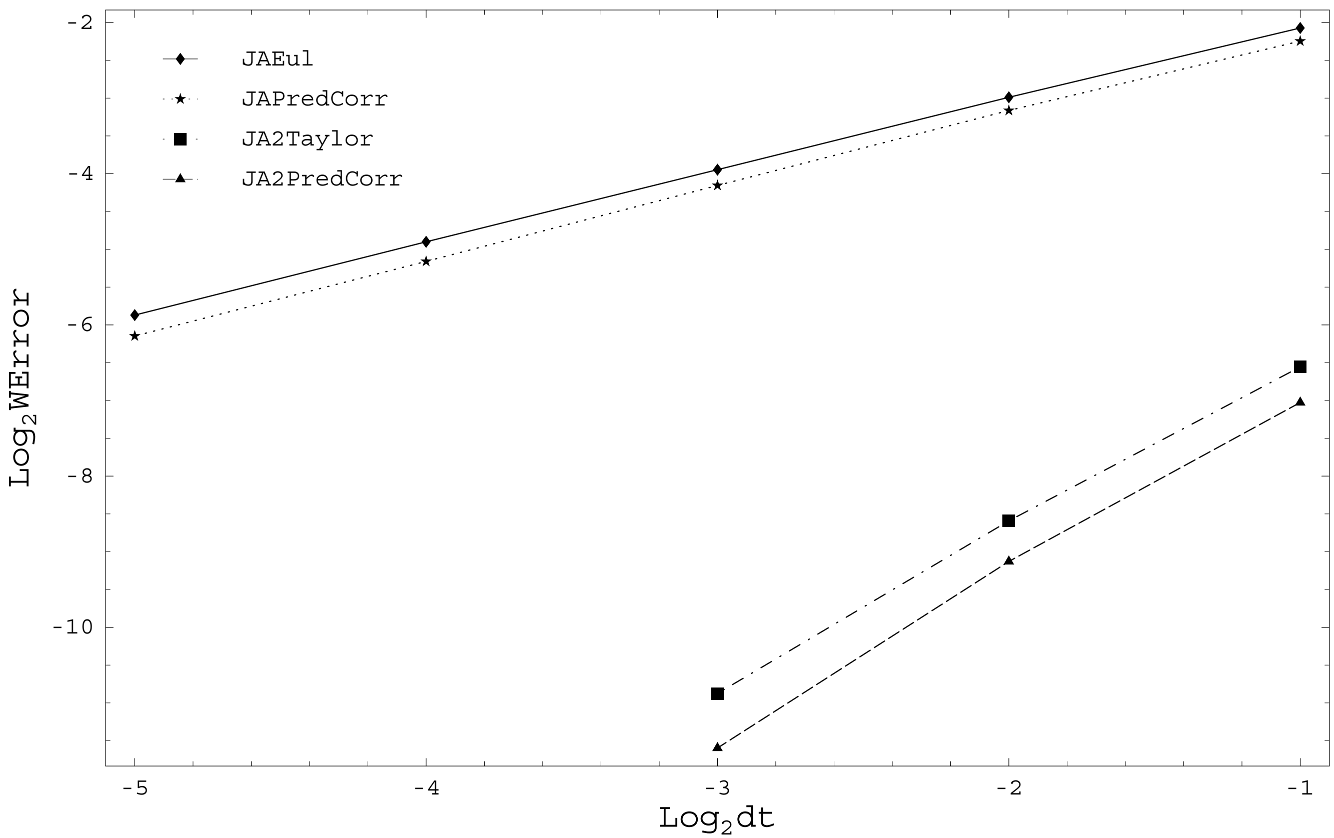
$$\varepsilon_w(\Delta) = \left|E\big((X_T - K)^+\big) - E\big((Y_N - K)^+\big)\right|$$

$$\mu = 0.05,\ \sigma = \kappa = \lambda = 0.2,\ X_0 = 100,\ T = 0.5,\ K = 125$$

- CPU times in seconds for 1m simulations with 256 time steps

JAEul	JAPC	JA2	JA2PC
46.6	48.7	48.0	50.3

Numerical Results (cont'd)



Log-log plot of weak error versus time step size.

Numerical Results (cont'd)

Relative Errors (%)

method \ dt	0.5	0.25	0.125	0.0625	0.03125
JAEuler	19.94	10.56	5.44	2.81	1.43
JAPredCorr	17.68	9.36	4.71	2.35	1.18
JA2Taylor	0.89	0.22	0.04		
JA2PredCorr	0.64	0.15	0.03		

Option price $= 1.19$

Conclusions

- **strong** vs **weak** schemes

- **strong** schemes: scenario analysis, filtering, hedge simulation

- **weak** schemes: evaluation of moments, derivatives, risk measures

- predictor-corrector $\Longrightarrow$ numerical stability $+$ efficiency

References

Hunter, C. J., P. Jäckel, & M. S. Joshi (2001). Getting the drift. *Risk* **14**(7), 81–84.

Joshi, M. & A. Stacey (2006). New and robust drift approximations for the LIBOR market model. Melbourne University, Technical report.

Kloeden, P. E. & E. Platen (1992). *Numerical Solution of Stochastic Differential Equations*, Volume 23 of *Appl. Math.* Springer.

Platen, E. (1995). On weak implicit and predictor-corrector methods. *Math. Comput. Simulation.* **38**, 69–76.