Risk Neutral and Actual Probabilities with an Endogenous Bankruptcy Jump-Diffusion Model

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Outline of the Talk

1. Bibliography
2. The Structural Approach to Liabilities and Default
3. A New Extended Framework
   with Unexpected Asset Value Shocks
4. Empirical Relevance of our Approach
5. Conclusion

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- Hilberink and Rogers [2002]
- Eom, Helwege and Huang [2003]

Bibliography

- Gerber and Shiu [1994]
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- Leland [2004]
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The Structural Approach to Liabilities and Default

The Black-Scholes-Merton Representations of Liabilities

Consider a Firm having the Balance Sheet :

Assets	Liabilities
A_0	E_0 (Equity) D_0 (Debt)

– Equity as a Call Option on the Assets

– Corporate Debt as a riskless bond and Short Put on the Assets

Company Structure and Firm Bankruptcy
The Merton's Viewpoint

The value of assets is monitored at debt maturity

If it is insufficient to cover the debt

Then, the company goes bankrupt

Company Structure and Firm Bankruptcy
The Black and Cox's Viewpoint

The value of assets is monitored *at any time*

As soon as the assets are not sufficient
to cover the engagements

The company bankrupts

Equity Maximization and Endogenous Criterion
The Leland (and Toft) Approach

Initial Value of the Firm as :

- Initial Value of Assets
- $+$ Discounted Value of Tax Advantages
- $-$ Discounted Value of Bankruptcy Costs

Bankruptcy threshold obtained endogenously through an Equity maximization criterion

The Hilberink and Rogers Framework

Merton, Black and Cox, Leland and Toft postulate
continuous assets dynamics

Hilberink and Rogers expanded the framework
with discontinuous assets dynamics

Their model is general and flexible,
it enables a large choice of debt maturity profiles

A New Extended Framework with Unexpected Asset Value Shocks

Limits of Past Approaches

Hilberink and Rogers concentrate on credit spreads

The recent paper by Dao and Jeanblanc using Kou processes also concentrates on credit spreads

We aim at studying *historical* default probabilities

–> Conceptual and Technical Challenge

The Problem

The key indicator we want to compute is :

$$P\left(\exists t \mid V_t < V_B\right)$$

This is the probability that the :

historical assets V cross

the endogenous risk-neutral bankruptcy barrier V_B

Need for a bridge between the real and risk-neutral worlds (equivalently the corporate and market finance worlds)

Our Answer...

...comes from the Insurance world

The Esscher transform, used by Gerber and Shiu, in the Transactions of the SOA

A tool to shift dynamics from a world to the other

Adaptation was needed with Kou processes : Important result given and proved in our paper

Based on this result, computations and empirical inferences become possible

Some technicalities

The assets price is an exponential of a Kou Process

$$V_t = V_0 exp\{X_t\} = V_0 exp\left(at + \sigma z_t + \sum_{k=1}^{N_t} Y_k\right)$$

– z is a standard Brownian motion
– N a Poisson process parameter (λ)
– Y a double exponential random variable

$$f_Y(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y>0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y<0\}}$$

– all these stochastic elements are independent

Market Values
Debt, Equity, Optimal barrier level

Using the arbitrage theory and results from Kou and Wang

- $\mathcal{D}$ debt value
- $\mathcal{E}$ equity value
- $\mathcal{V}$ firm value
- V_B Optimal barrier level
- are obtained in closed form formulae

$$V_B = \frac{\frac{C+mP}{r+m}\beta_{1,r+m} - \frac{\theta C}{r}\beta_{1,r}}{1 + \alpha\beta_{1,r} + (1-\widehat{\alpha})\,\beta_{1,r+m}}$$

Changing Universe

If the assets price is given by an exponential of a Kou process X in the historical universe and if we choose as EMM the Esscher measure Q_h such that

$$\left(\frac{dQ_h}{dP}\right)_t = \frac{e^{hX_t}}{E_P[e^{hX_t}]}$$

the parameter h chosen such that the discounted prices are martingales w.r.t Q_h then

the process X remains a Kou process in this risk neutral universe

Changing Universe formulae

$$\left\{\begin{array}{l} \widehat{p} = \frac{p\eta_1}{\zeta(\eta_1-h)} \\ \widehat{q} = 1 - \widehat{p} \\ \widehat{\eta_1} = \eta_1 - h \\ \widehat{\eta_2} = \eta_2 + h \\ \widehat{\lambda} = \lambda\zeta \\ \zeta = \frac{p\eta_1}{\eta_1-h} + \frac{q\eta_2}{\eta_2+h} \end{array}\right. \tag{1}$$

Historical Probabilities

$$\int_0^\infty e^{-\rho t} P\left(\tau_l \le t\right) dt = \frac{1}{\rho} \int_0^\infty e^{-\rho t} dP\left(\tau_l \le t\right) = \frac{1}{\rho} E_P\left[e^{-\rho \tau_l(X)}\right] \quad (2)$$

where it is known since Kou and Wang that :

$$E_P\left[e^{-\rho \tau_l(X)}\right] = \frac{\eta_2 - \beta_{3,\rho}}{\eta_2} \frac{\beta_{4,\rho}}{\beta_{4,\rho} - \beta_{3,\rho}} e^{l\beta_{3,\rho}} + \frac{\beta_{4,\rho} - \eta_2}{\eta_2} \frac{\beta_{3,\rho}}{\beta_{4,\rho} - \beta_{3,\rho}} e^{l\beta_{4,\rho}}, \quad (3)$$

where $-\beta_{3,\rho}$ and $-\beta_{4,\rho}$ are the two negative roots of the following Laplace exponent function $G(\cdot)$:

$$G^P\left(\beta\right) = \frac{1}{2}\sigma^2\beta^2 + a\beta + \lambda^P \left(\frac{p^P \eta_1^P}{\eta_1^P - \beta} + \frac{q^P \eta_2^P}{\eta_2^P + \beta} - 1\right) \quad (4)$$

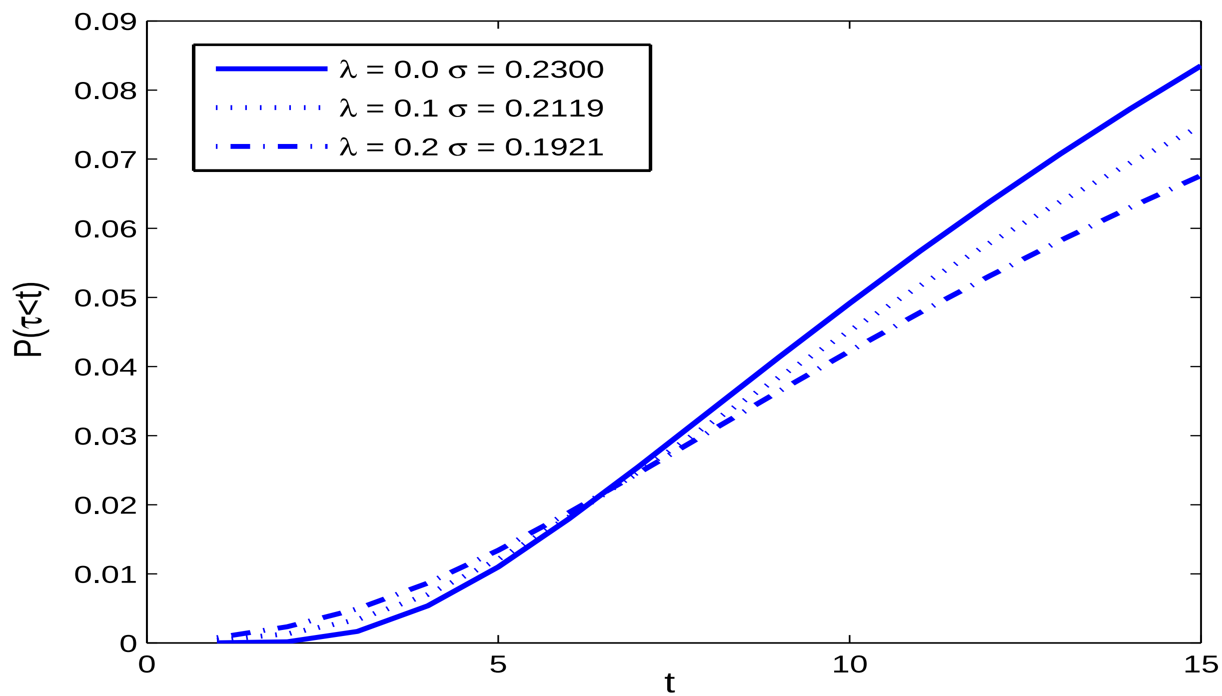
Empirical Relevance of our Approach

Traditional approaches *do* predict long-term historical default probabilities (see empirical overview by Leland [2004])

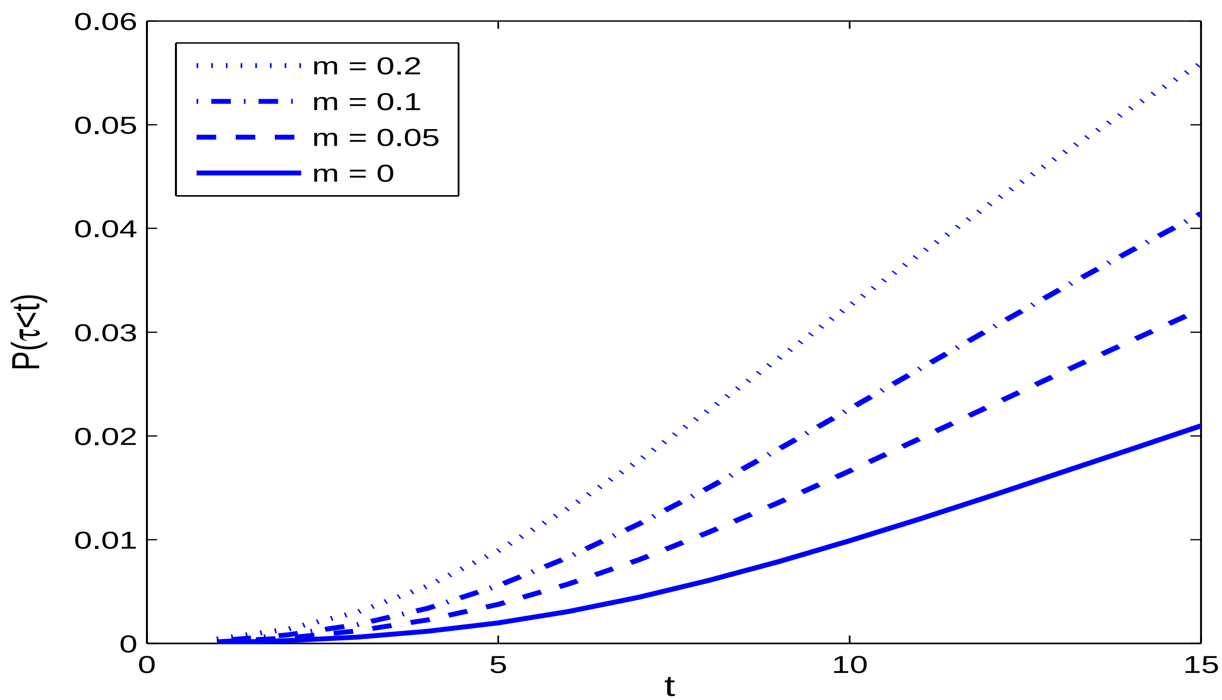
They *do* not predict correctly short/medium-term historical default probabilities

On the contrary, our approach, which incorporates unexpected discontinuities, allows for a full estimation and prediction of default probabilities

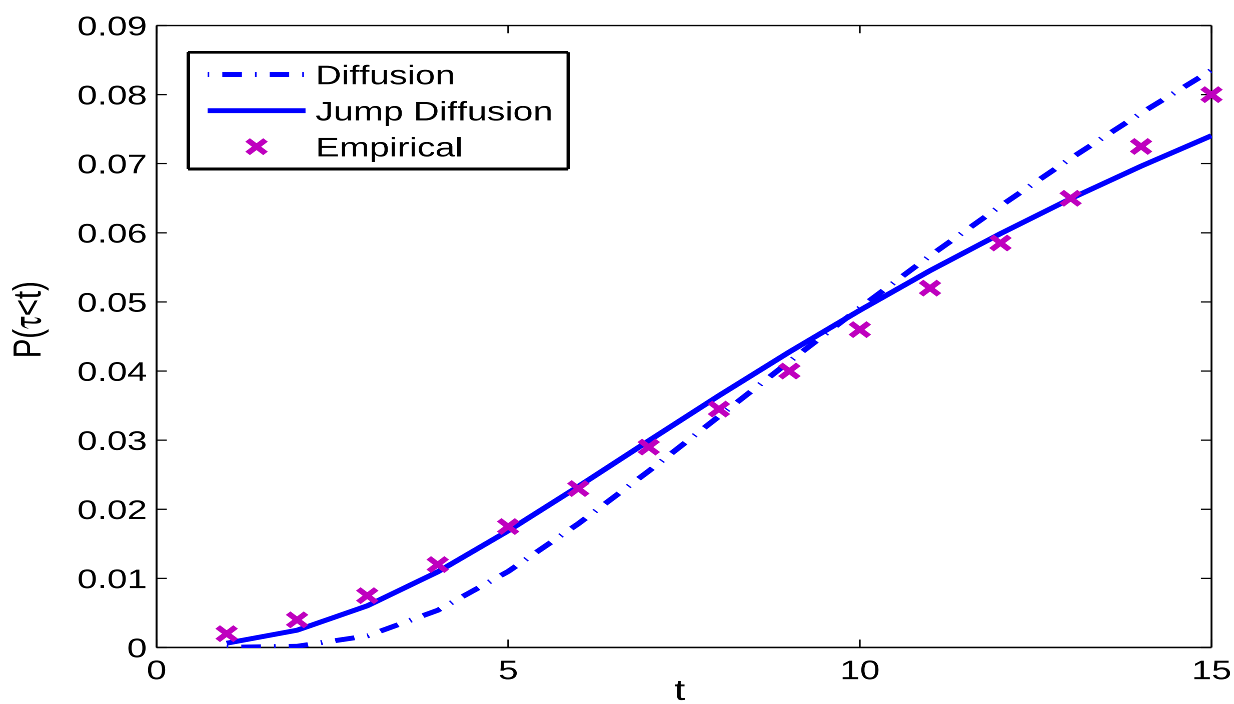
Cumulative Default Probabilities Introducing Discontinuities



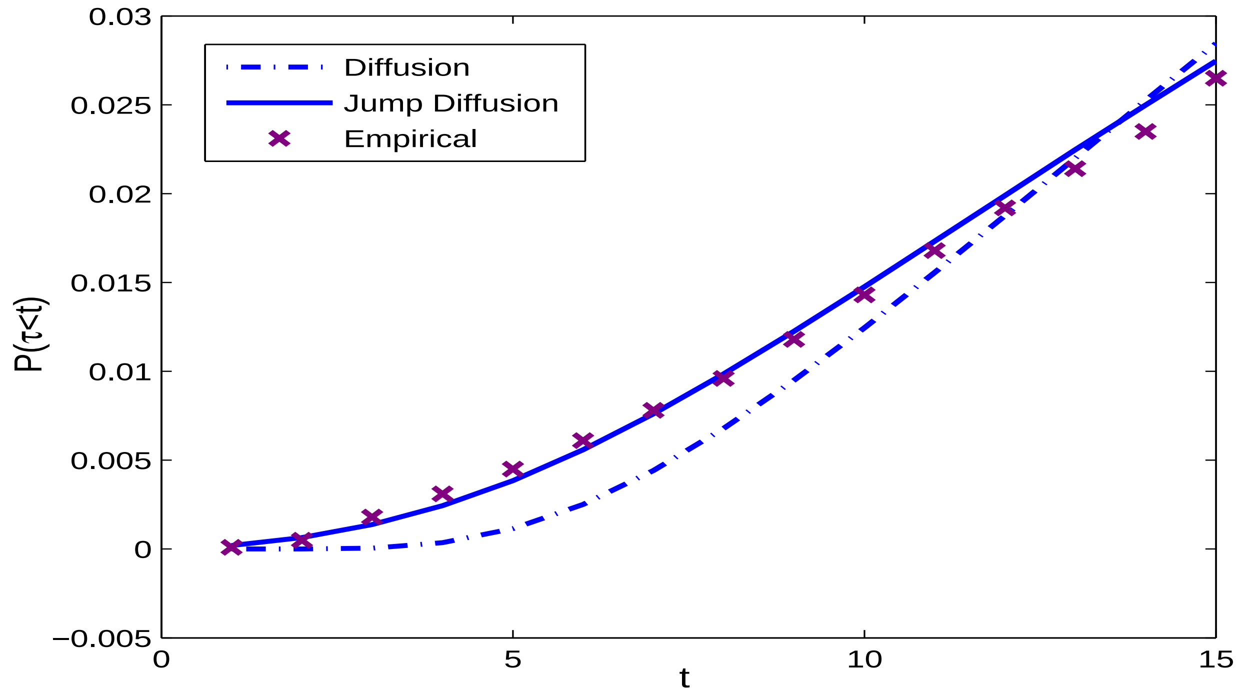
Cumulative Default Probabilities
Effect of Debt Maturity



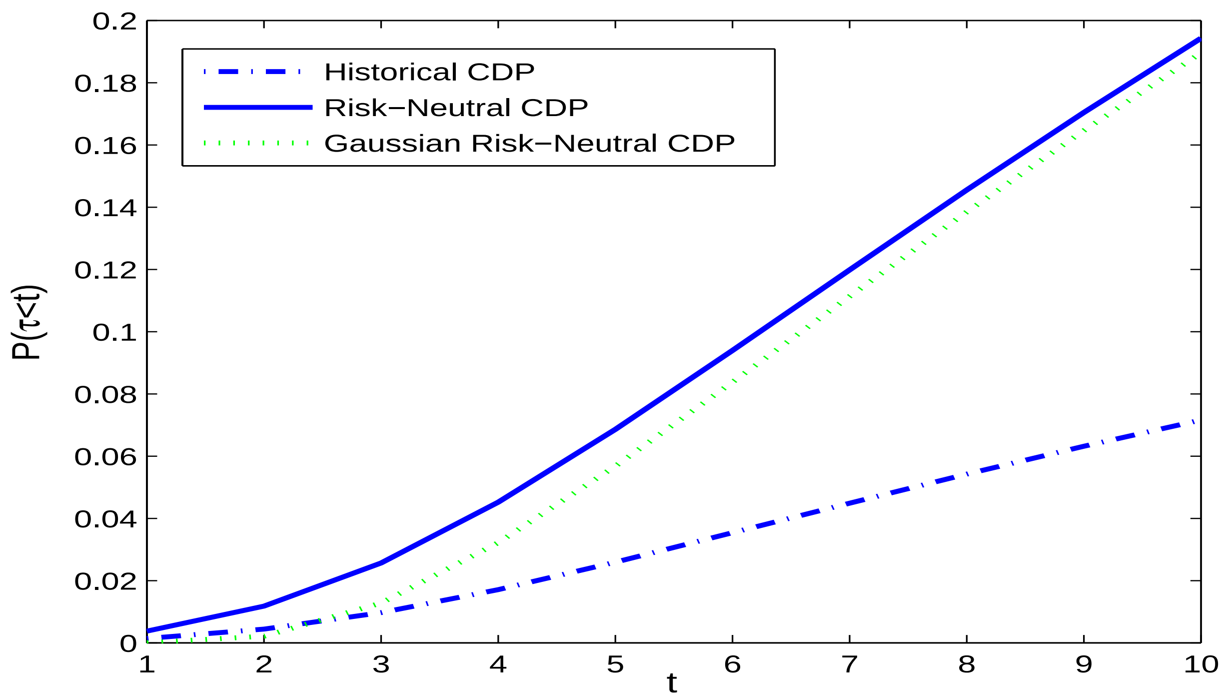
Comparison with Empirical Data
Baa-Rated Bonds



Comparison with Empirical Data
A-Rated Bonds



Historical vs Risk-Neutral Default Probabilities



Conclusion

A problem at the junction of corporate and market finance

A new way to compute default probabilities

Theoretically grounded and empirically relevant

Forthcoming : deeper empirical analysis
and larger tests of predictions