# *Pricing Equity Default Swaps*

Oliver Chen

Centre for Financial Engineering and Department of Mathematics,

National University of Singapore

`ochen@nus.edu.sg`, `http://www.math.nus.edu.sg/~ochen/`

Quantitative Methods in Finance 2005, 14 December.

# Outline

I. Description of equity default swaps (EDSs)
II. Pricing with a CEV equity model
III. The case for using a credit-equity model
IV. Pricing with a credit barrier model
V. Further extensions of credit barrier models
VI. Conclusions

# *Based on:*


- Albanese, C. and O. Chen (2005). *Discrete credit barrier models*. Quantitative Finance, **5(3)**, 247–256.
- Albanese, C. and O. Chen (2005). *Pricing equity default swaps*. RISK, **18(6)**, 83–87.
- Albanese, C., O. Chen and A. Dallesandro (2005). *Dynamic credit correlation modeling*. Submitted.

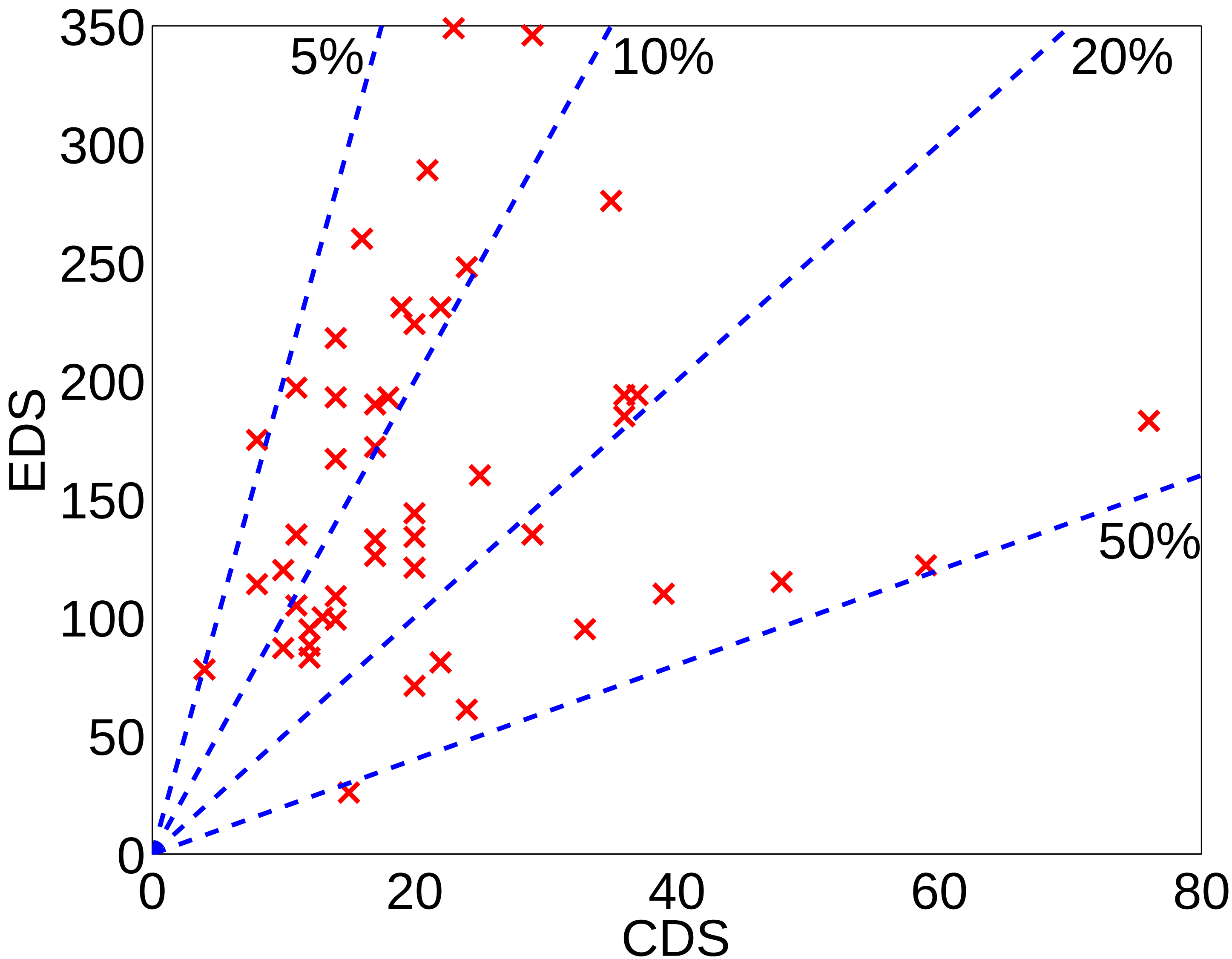
# *I. Description of EDSs*

- Similar to a deep out-of-the-money digital American put.
- Payout occurs if share price declines below a predetermined level (typically, $30\%$ of the share price at issue of EDS). This is referred to as equity default.
- Payment is not up-front, but occurs periodically (typically semi-annually) until the earlier of EDS maturity or equity default.

# *CDS vs. EDS*

| | **Credit default swap (CDS)** | **Equity default swap (EDS)** |
|---|---|---|
| Payments | semi-annual | semi-annual |
| Default trigger | Credit event | Stock trades under 30% of the original price |

# EDS vs. CDS rates for Eurostoxx 50, 04/08/2004




Data courtesy of JP Morgan

## II. Pricing with a CEV equity model

- A process $S_t$ is called a CEV diffusion if it solves the SDE:

$$dS_t = \mu S_t dt + \sigma S_t^{\beta+1} dW_t$$

where $W_t$ is standard Brownian motion.

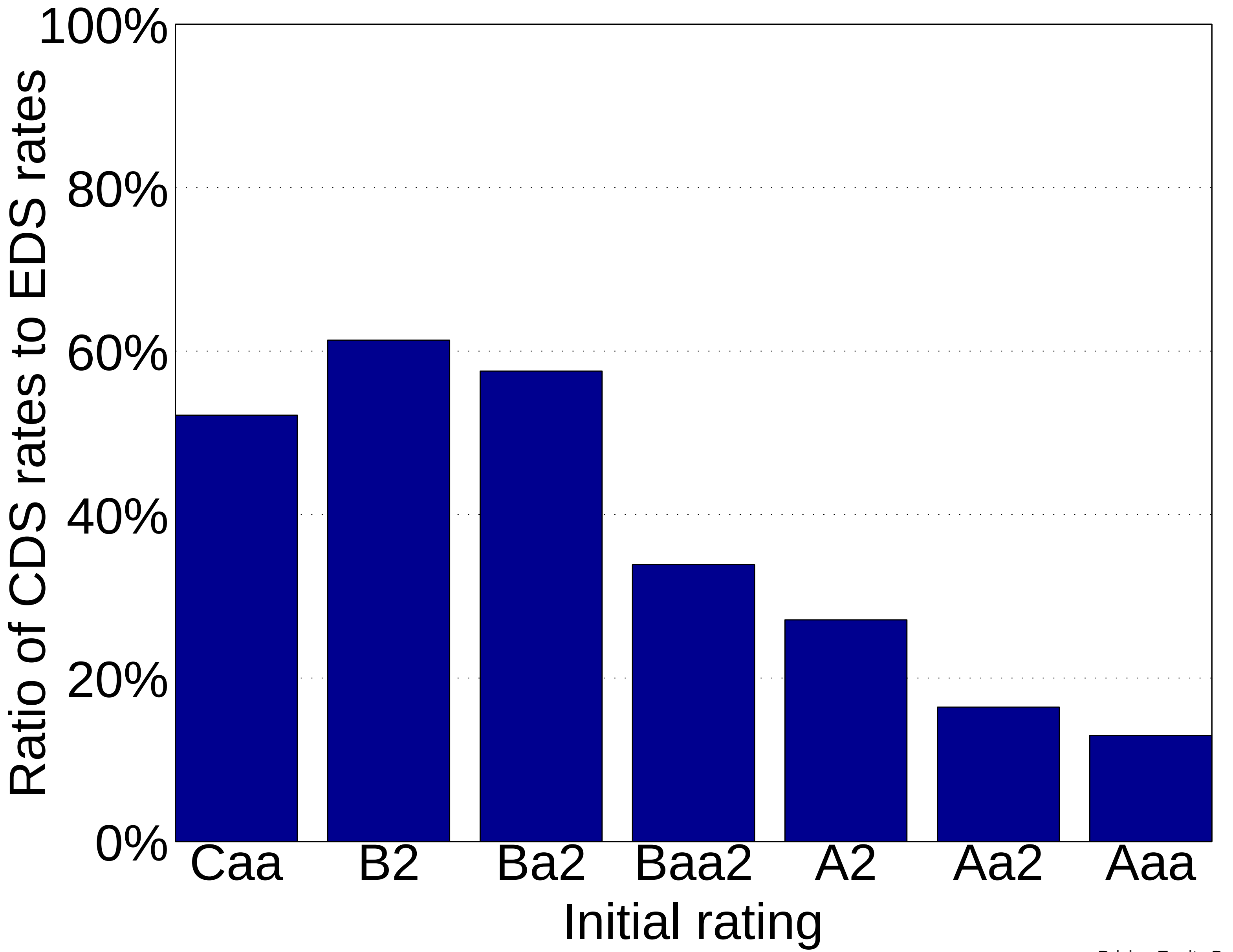
- For $\beta < 0$, there is a non-zero probability of the process becoming zero (and staying there).

- Thus, the CEV process is suitable to compare EDS and CDS rates if we consider a zero share price as credit default.

# Calibrating the CEV model

- It is possible to calculate the probability that a CEV process will be below a certain level within a certain time, and also the probability that a CEV process will be zero within a certain time.
- Thus, EDS and CDS rates can be computed.
- $\sigma$ and $\beta$ can be adjusted to match a CDS rate and an ATM implied volatility.
- Once $\sigma$ and $\beta$ are determined, the EDS rate can be computed.

# CDS to EDS rates for the CEV process



## *III. The case for using a credit-equity model*

- With such a large drop in share price needed to trigger equity default, there are likely to be accompanying declines in the credit quality.
- Capital structure of the firm will likely be changed, so one needs to take into account the credit situation.
- A pure equity model may not be sufficient to price EDSs.

# *IV. Pricing with a credit barrier model*

- Goal: to specify a stochastic process for credit quality.
- Characteristics of credit quality processes for calibration:
  - credit rating transition probabilities in the statistical measure
  - credit default probabilities in the statistical measure
  - credit default probabilities in the pricing measure

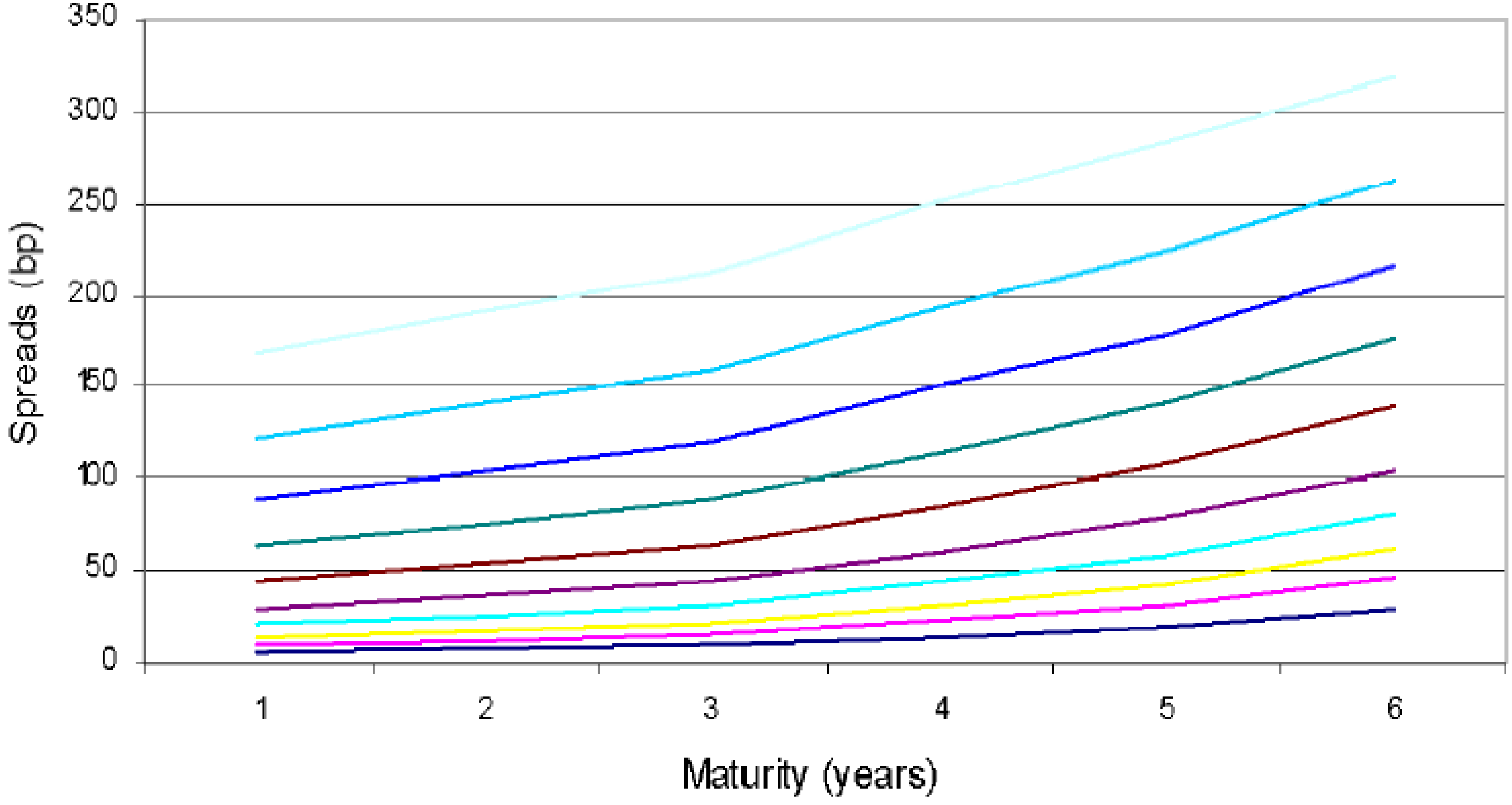
# *Mapping credit quality to equity*

- Once the credit quality process is specified, we can construct a one-to-one mapping to share price.
- This makes the assumption that the equity price is a deterministic function of the credit quality.
- With this, it is possible to price EDSs.

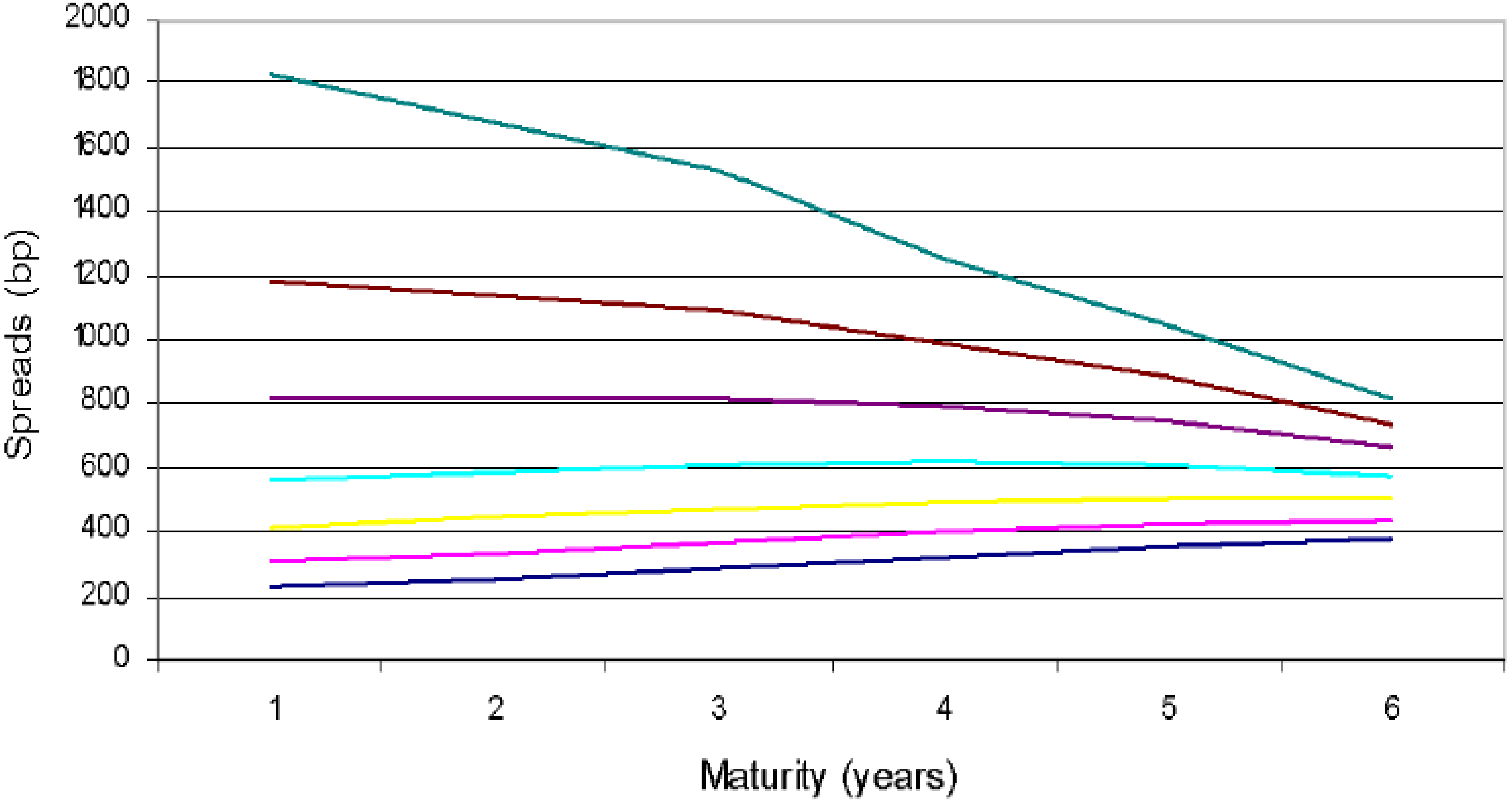
# Optimisation target for statistical measure

| % | Aaa | Aa | A | Baa | Ba | B | C | Default |
|---|---|---|---|---|---|---|---|---|
| **Aaa** | 91.8 | 7.4 | 0.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Aa** | 1.2 | 90.7 | 7.7 | 0.3 | 0.1 | 0.0 | 0.0 | 0.0 |
| **A** | 0.1 | 2.5 | 92.0 | 4.8 | 0.5 | 0.1 | 0.0 | 0.0 |
| **Baa** | 0.1 | 0.3 | 5.5 | 88.5 | 4.7 | 0.7 | 0.1 | 0.2 |
| **Ba** | 0.0 | 0.0 | 0.5 | 5.6 | 85.4 | 6.7 | 0.5 | 1.3 |
| **B** | 0.0 | 0.0 | 0.1 | 0.4 | 6.7 | 83.4 | 2.6 | 6.8 |
| **Caa-C** | 0.0 | 0.0 | 0.0 | 0.6 | 1.6 | 4.1 | 68.0 | 25.6 |

# CDS spreads: Investment grade



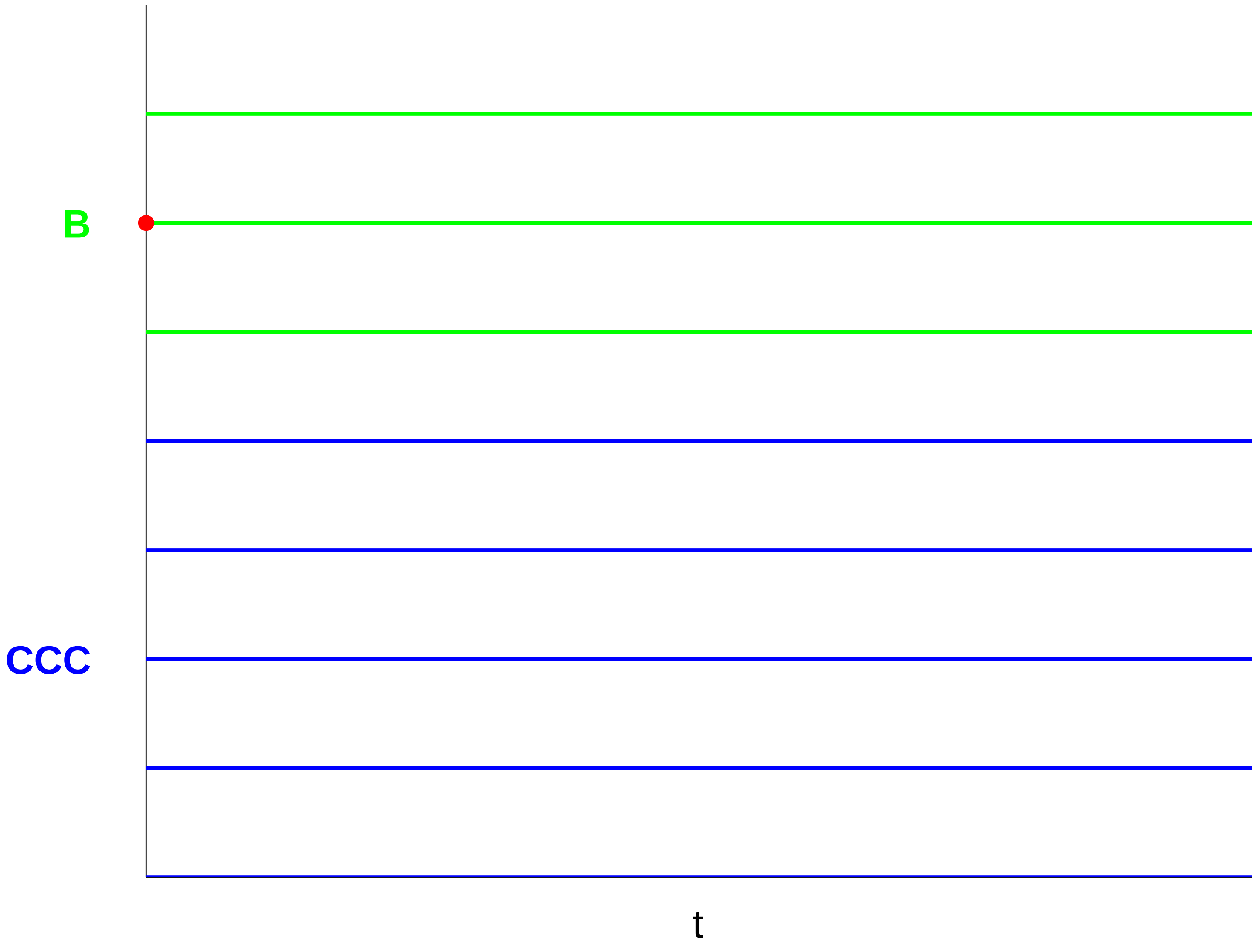
# CDS spreads: Speculative grade



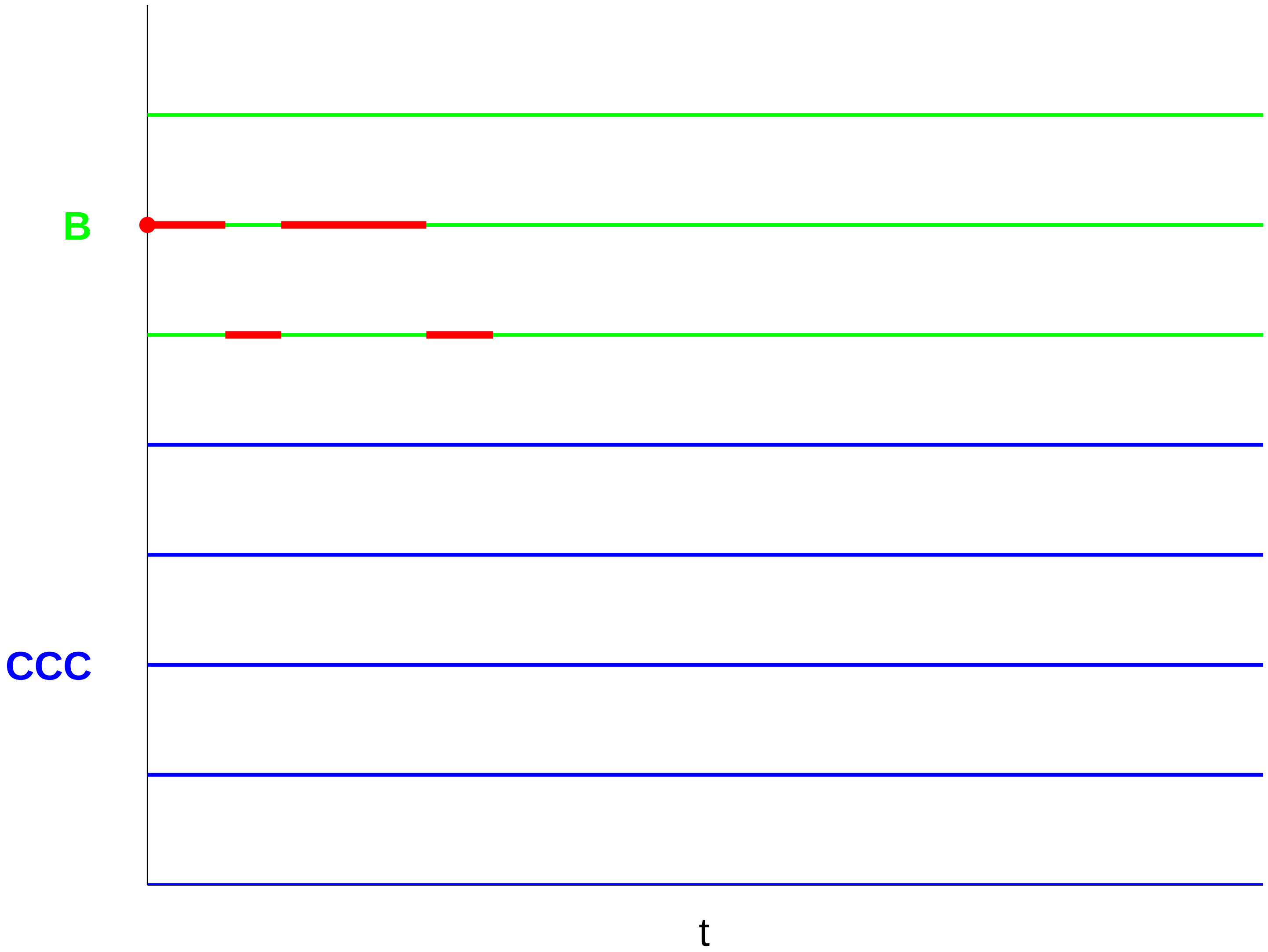
# *The discrete stochastic process*

- One can specify the credit quality process as a continuous stochastic process, but the complexity required would make calculations numerically intensive.
- Instead of a diffusion process, we use a birth and death process on a finite, discrete lattice.
- Integrals become finite sums and application of a stochastic time change adds no numerical overhead.

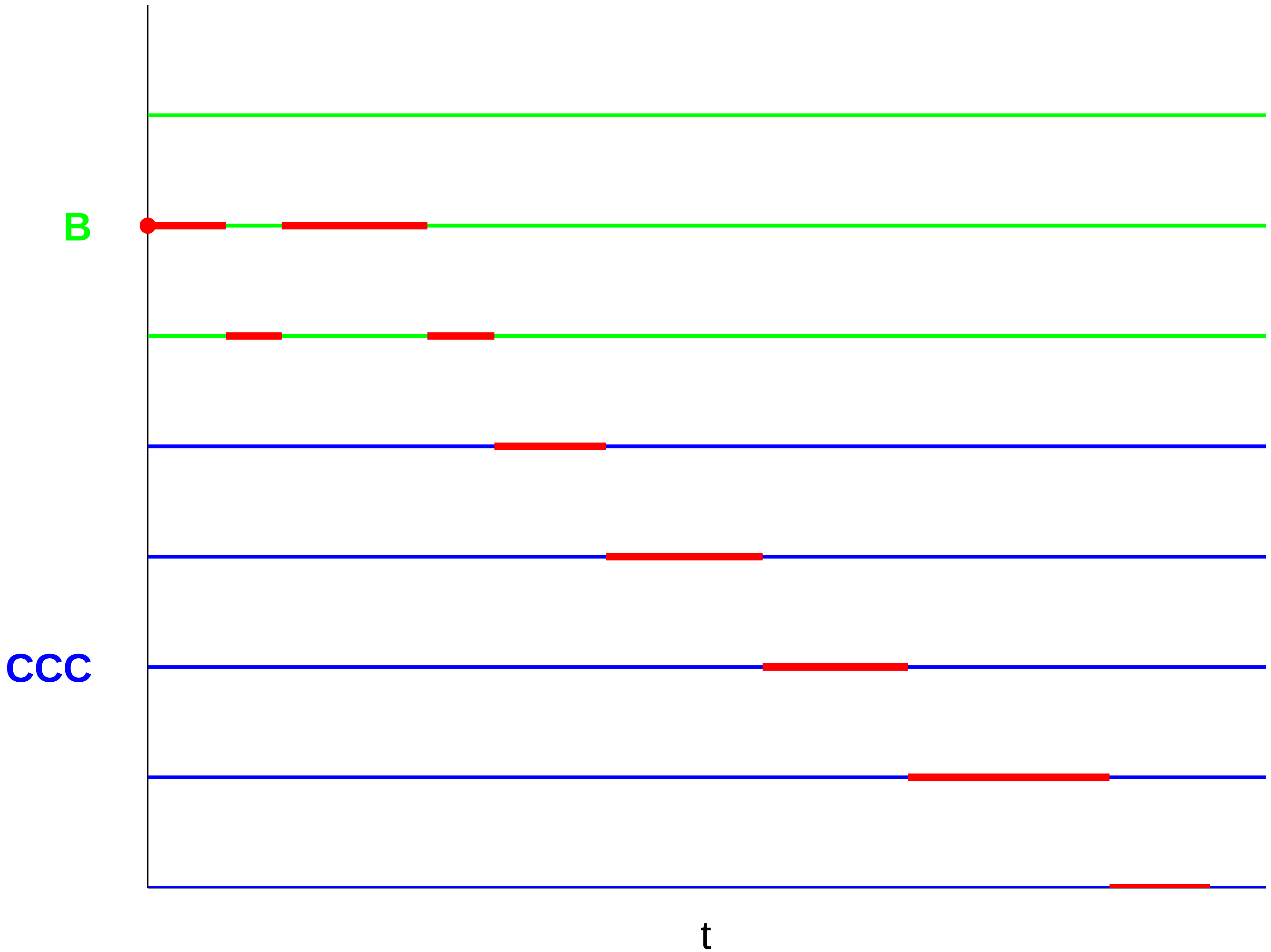
# A sample credit process



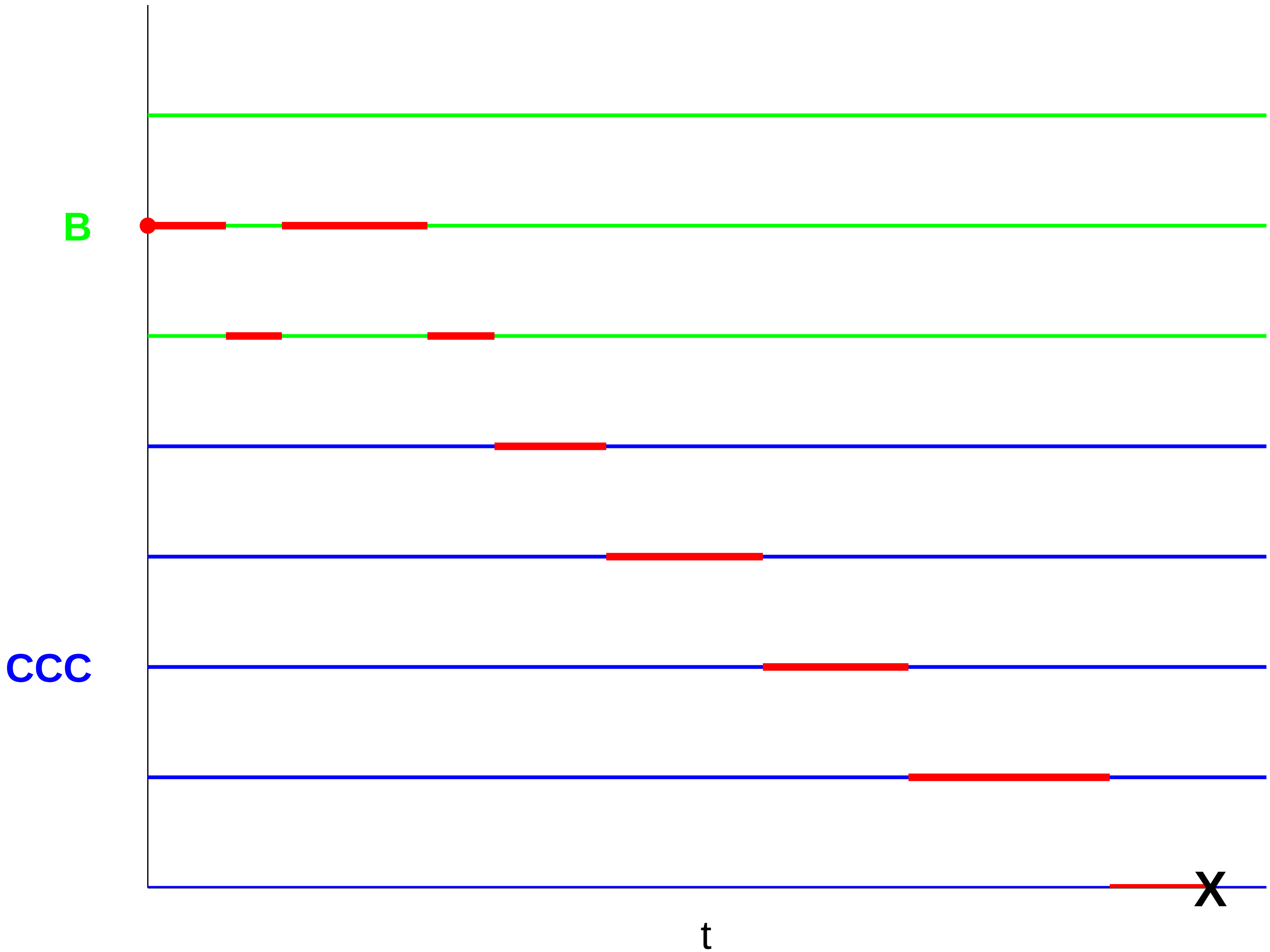
# A sample credit process



# A sample credit process



# A sample credit process



# *Lattice method*

- Distinct from traditional trinomial or binomial tree methods.
- State space is discretised, but time is continuous.
- Long-time transition probabilities can be computed exactly with the same computational cost as for shorter times.

# Birth-death processes

- The usual random walk processes are discrete state, discrete time approximations to Brownian motion.
- Birth-death processes are continuous time limits of random walks.
- In a birth-death process, only nearest neighbour transitions are allowed.

# Birth and death rates

- We specify a general, stationary birth-death process $x_t$ on a finite lattice $\tilde{\Lambda} = \{0, \ldots, N\}$

- The arrival rate of transitions are Poisson processes, so that as $t \to 0$, we have:

$$
\begin{aligned}
p(t; i, i+1) &= A_i t + o(t) && 0 \le i \le N-1 \\
p(t; i, i-1) &= C_i t + o(t) && 1 \le i \le N \\
p(t; i, i) &= 1 - (A_i + C_i) t + o(t) && 0 \le i \le N
\end{aligned}
$$

- Here, $C_0 = 0$, $A_N = 0$, $C_1, \ldots, C_N > 0$, $A_1, \ldots, A_{N-1} > 0$ and $A_0 = 0$ so that $0$ is an absorbing state.

# Infinitesimal generator

- Defining $B_i = -(A_i + C_i)$ for $i = 0, \ldots, N$, the matrix is tri-diagonal and given by:

$$\mathcal{L} \;=\; \begin{pmatrix} B_0 & A_0 & 0 & \cdots & & & \\ C_1 & B_1 & A_1 & 0 & \cdots & & \\ 0 & C_2 & B_2 & A_2 & & & \\ \vdots & & \ddots & \ddots & \ddots & & \\ & & & C_{N-1} & B_{N-1} & A_{N-1} \\ & & & & C_N & B_N \end{pmatrix}$$

- The matrix $\mathcal{L}$ is called the infinitesimal generator of the process $x_t$.

# Probability kernel

- The probability kernel is given by:

$$p(t; i, j) \;=\; e^{t\mathcal{L}}(i, j).$$

- If $\mathcal{L}$ is diagonalizable, then this exponential can be computed efficiently in terms of the eigenvalues and eigenvectors of $\mathcal{L}$.

# Specifying $\mathcal{L}$

- We can specify the local volatility and drift by hand:

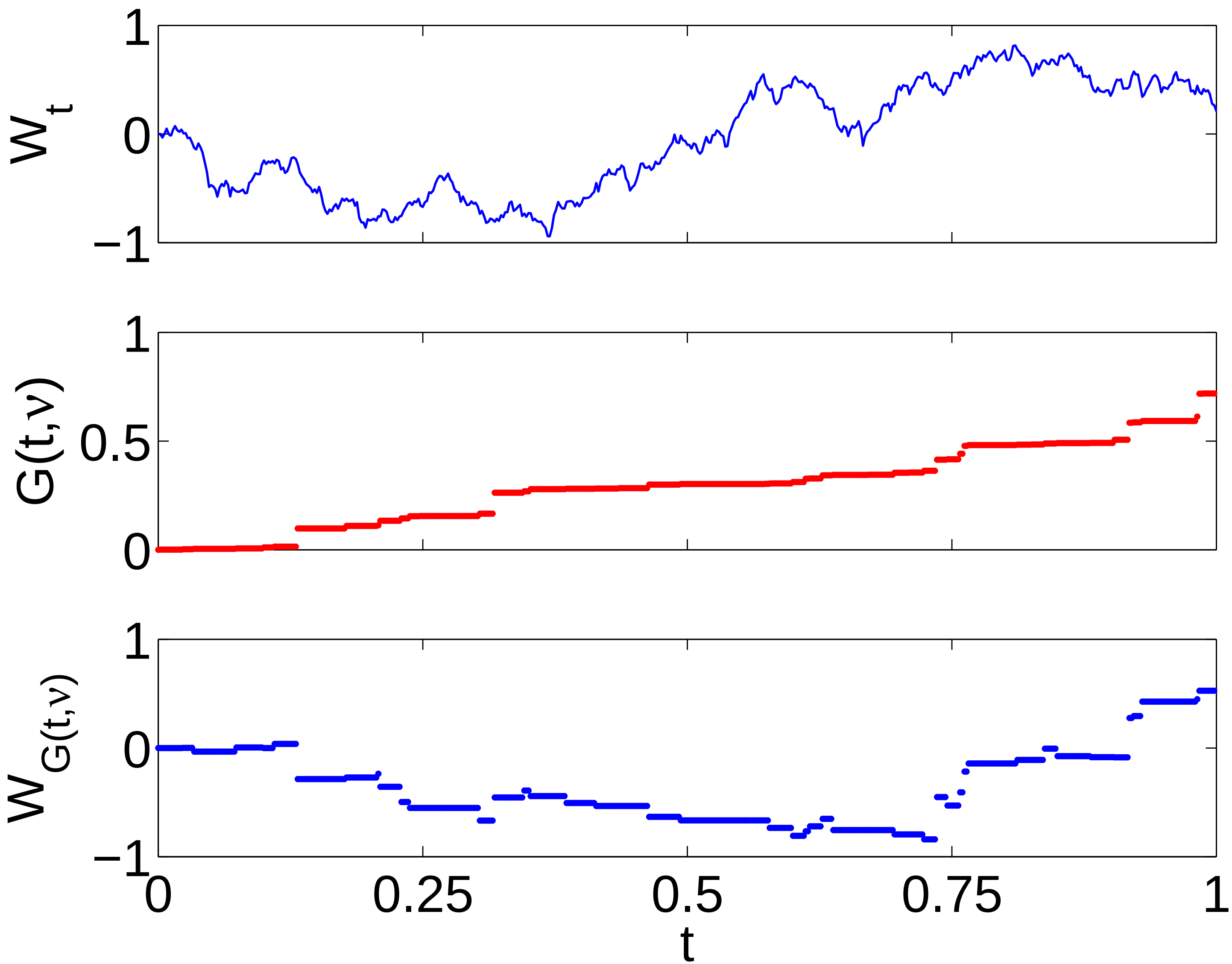
$$
\begin{aligned}
\mathcal{L}(x, x+1) - \mathcal{L}(x, x-1) &= \mu(x) \\
\mathcal{L}(x, x+1) + \mathcal{L}(x, x-1) &= \sigma(x)^2 \\
\mathcal{L}(x, x+1) + \mathcal{L}(x, x) + \mathcal{L}(x, x-1) &= 0
\end{aligned}
$$

- We use different $\mu$ for the statistical and pricing measures.

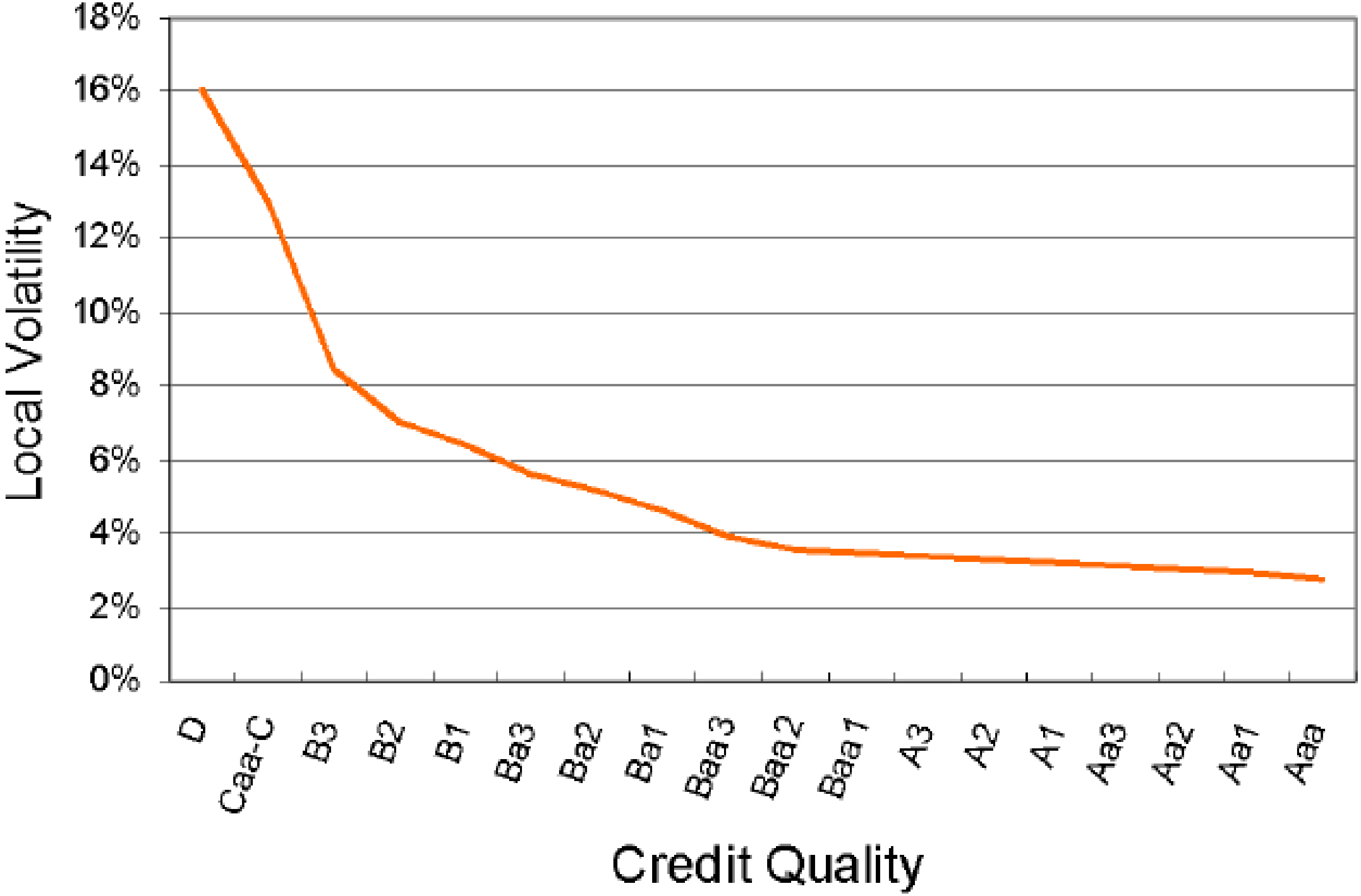
# *Adding jumps*

- With just these birth-death processes, we were unable to calibrate to historical data. Jumps needed to be added.
- Jumps can be added by subordinating on monotonic stochastic processes.
- The transition probabilities with jumps is calculated from integrating the transition probabilities without jumps against the distribution of the time change.
- This is just a Laplace transform, with an analytical formula if the time change is a gamma process.

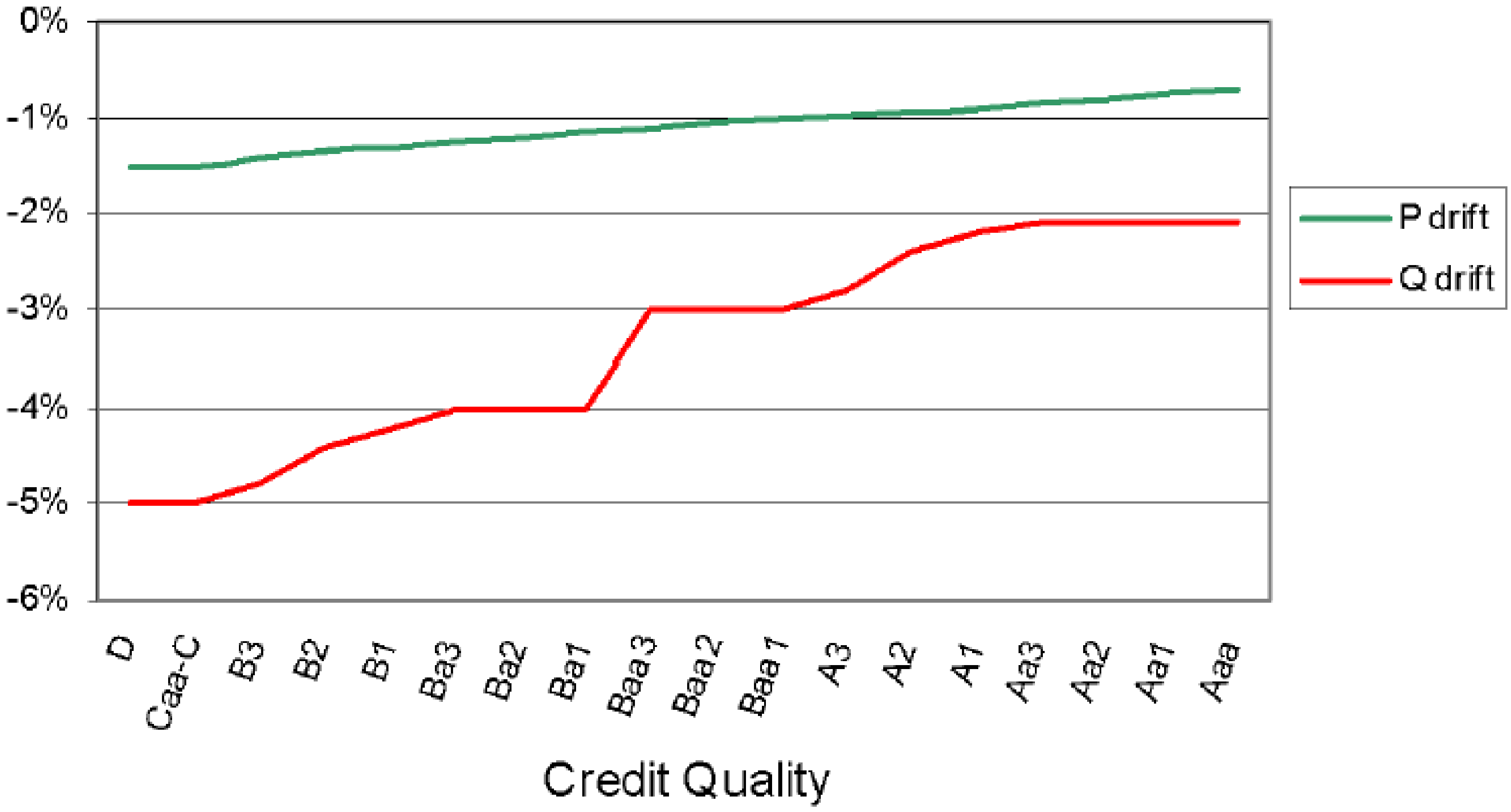
# Stochastic time change



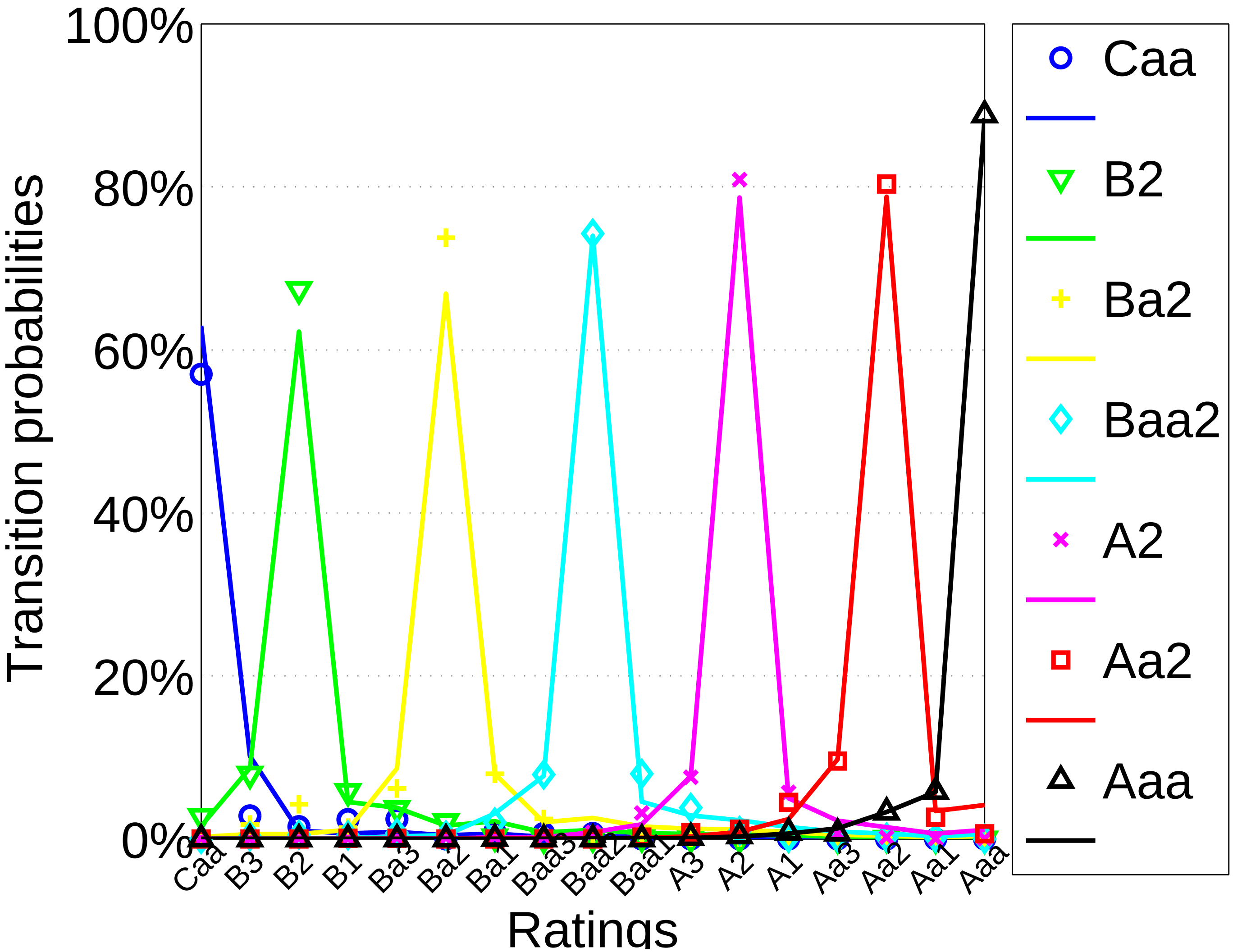
# Local volatility



# Drifts

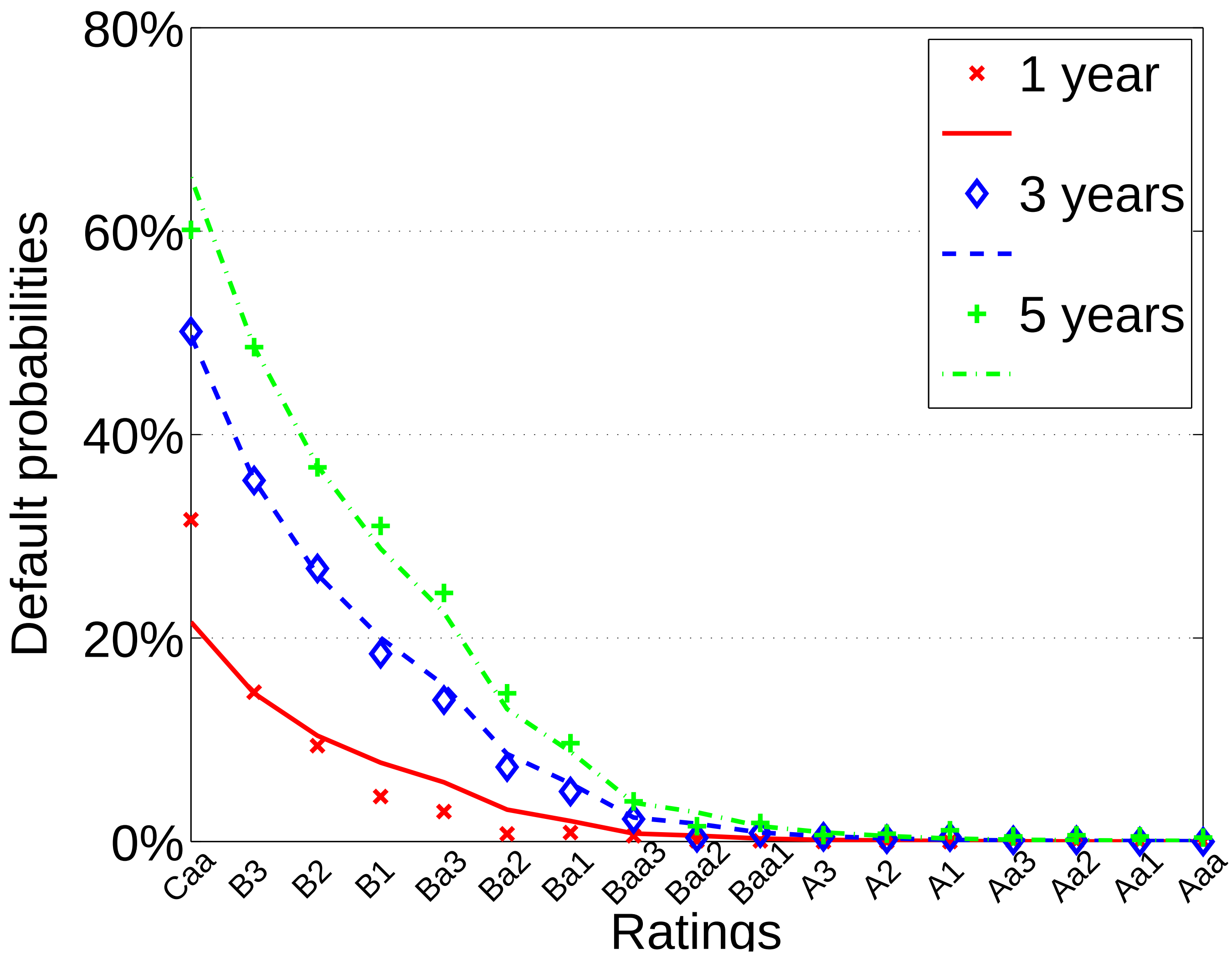

0%
-1%
-2%
-3%
-4%
-5%
-6%
P drift
Q drift
D
Caa-C
B3
B2
B1
Ba3
Ba2
Ba1
Baa 3
Baa 2
Baa 1
A3
A2
A1
Aa3
Aa2
Aa1
Aaa
Credit Quality

# *Historical transition probabilities*



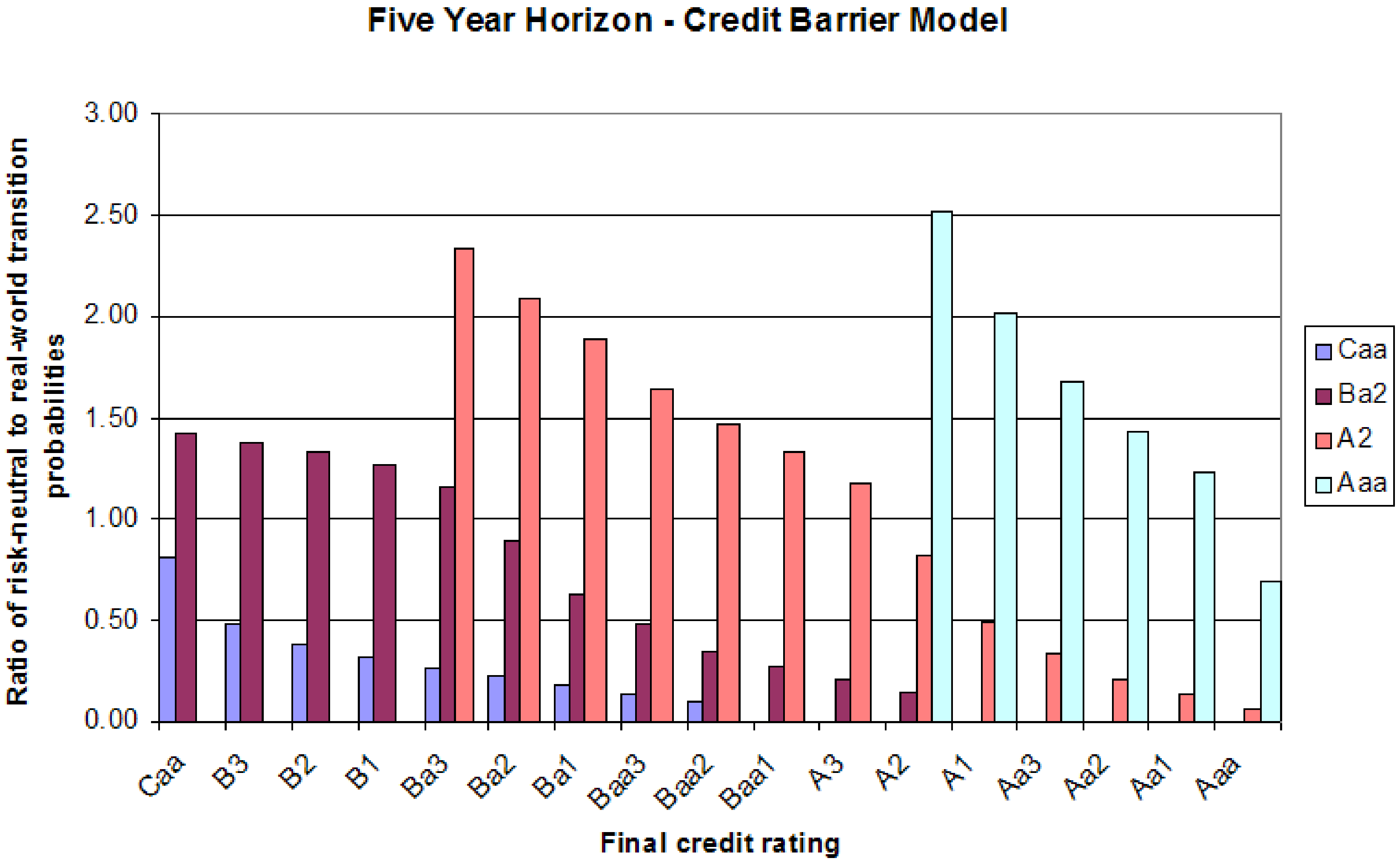

Lines for model values, markers for empirical values.

# Historical default probabilities

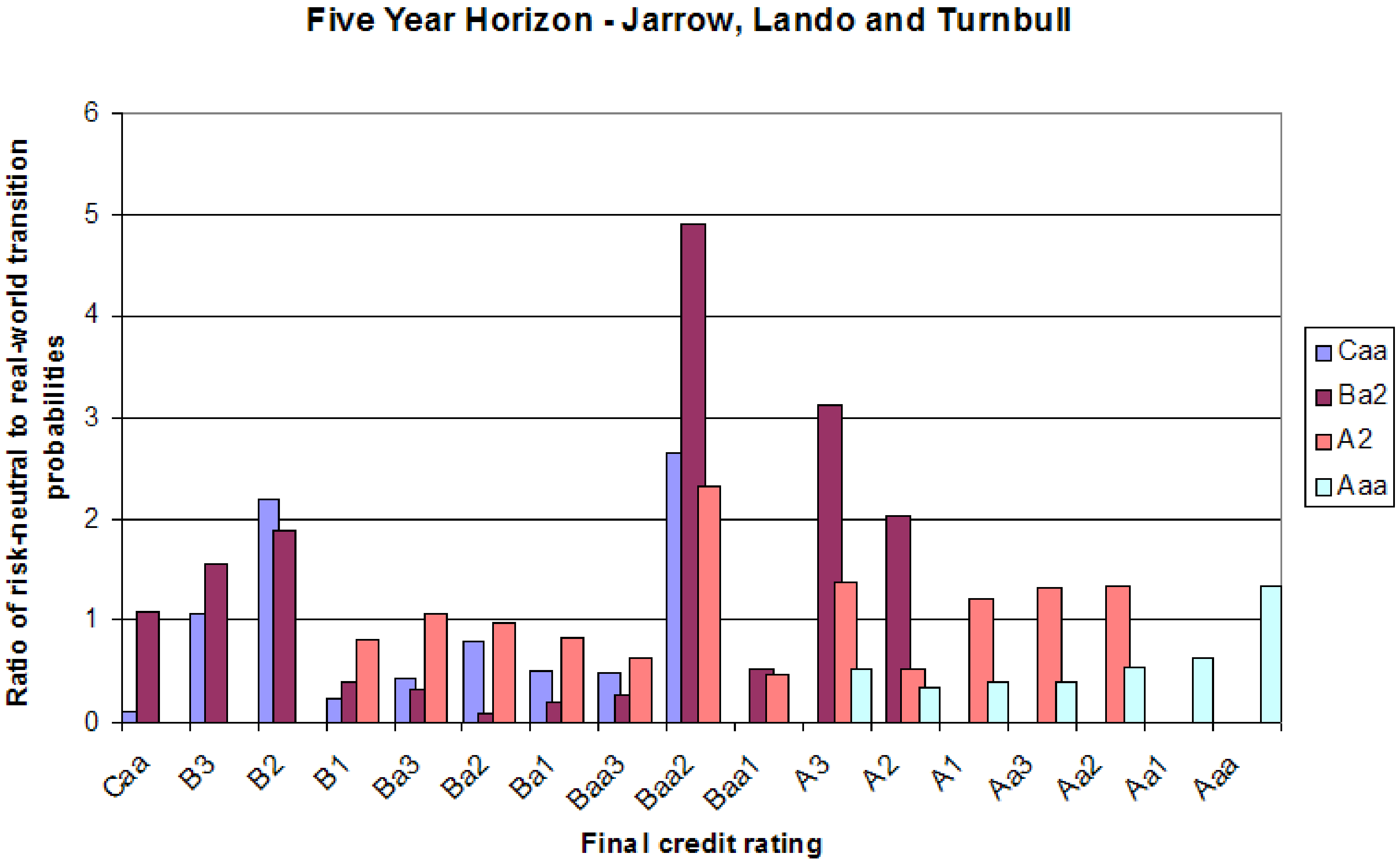



Lines for model values, markers for empirical values.

# Risk-neutral transition probabilities



# Risk-neutral transition probabilities



# Mapping to equity

- The credit quality $x_t$ is mapped to equity prices via a deterministic, monotonic function $\Phi$ at some horizon date $T$:

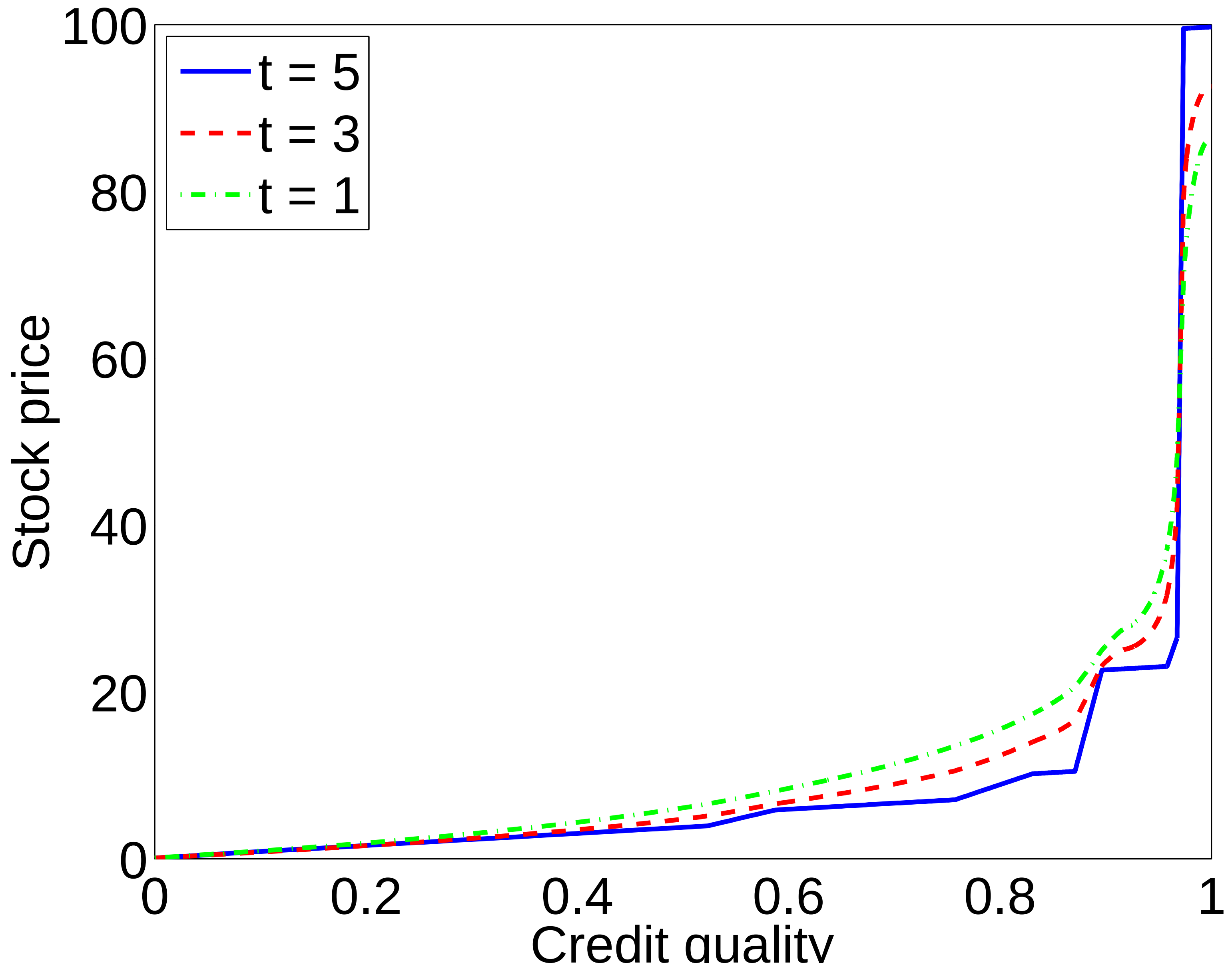
$$S_T(x_T) \;=\; e^{rT}\Phi(x_T)$$

- For $t_i < T$, we take the discounted expectation of $\Phi$:

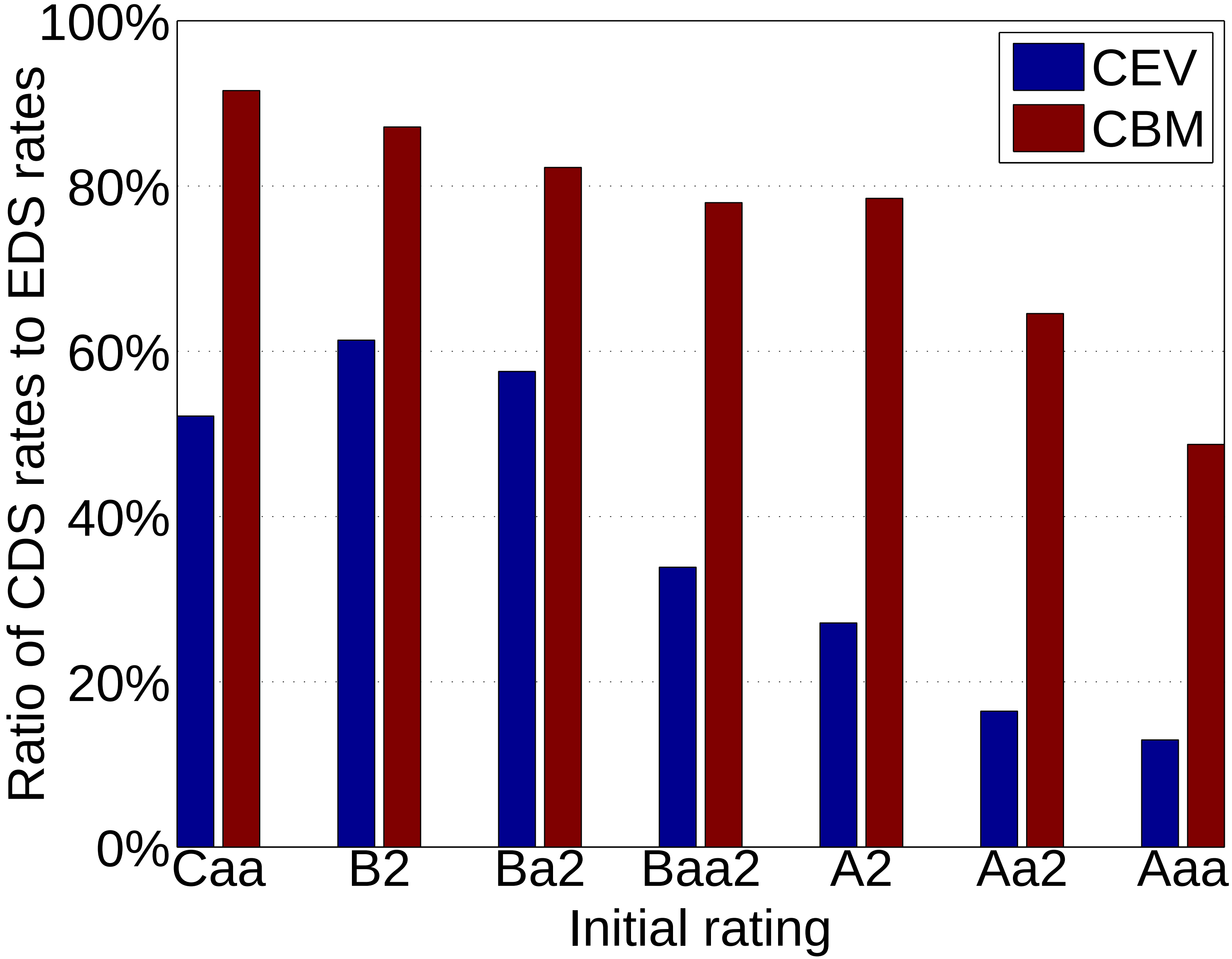
$$S_{t_i}(y) \;=\; e^{rt_i}\mathbb{E}[\Phi(y_T)|y_{t_i}]$$

- $\Phi$ is specified so that a given drop in equity price corresponds to a given drop in credit quality.

# Credit to equity mapping



# *CDS to EDS rates for the CBM*



# *V. Further extensions of credit barrier models*

- Multi-name credit instruments can be priced if correlation is taken into account.
- The approach that we take is to correlate on a medium-period (1 yr.) basis to a macroeconomic index variable.
- The correlation of each reference name to the index is dependent on the credit quality of the entity.
- Computation in this framework is efficient.

# *VI. Conclusions*

- Equity default swaps were priced with a CEV equity model.
- They were also priced using a credit barrier model to take into account credit considerations.
- Significant differences in EDS rates found using the two methods were found. The possibility of jump-to-default has a large effect on the CDS to EDS ratio.