Real Option Theory and Electricity Forwards

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Motivation and General Context

- National Electricity Market liberalized (deregulated)
- Wholesale electricity prices are determined by intersecting demand and supply every 5 min.
- Very high volatility ($40/MWh to $10,000/MWh) over few hours
- Retailer buys at a floating price and sells at a fixed price: huge market risk
- Need for hedge : Electricity forwards
- Question: What is the fair price of electricity forwards?

Pricing Methodologies

- In the literature: different methods for pricing electricity forwards
  - No-arbitrage pricing
  - Equilibrium pricing

In this paper:

- Applying Real option theory to electricity, because it has similar characteristics to real asset
  - Not tradeable
  - Not storable

- In Real option pricing literature, there are two main approaches:
  - Replicating the non-tradeable asset by a portfolio of tradeable assets
  - Applying some new principals with roots in the actuarial sciences (Musiela, Henderson, McCradle..)

- This paper applies a new approach by Elliott and van der Hoek (2003), a generalization of the two approaches

- Let X(t) denote the investor's wealth at time t and

$$V_0(x) = \max \mathsf{E}[u(X(1))]$$

where the maximum is taken over all portfolios in tradeable assets with $X(0) = x$ and they have uncertain value $X(1)$ in one period's time. We also let

$$V_G(x) = \max \mathsf{E}[u(X(1) + G)]$$

where G is the value of the contingent claim at time 1 and the maximum is taken over all portfolios in tradeable assets with $X(0) = x$.

- We define $\pi^b(G)$, the bid price of G, as the solution of

$$V_G(x - \pi) = V_0(x)$$

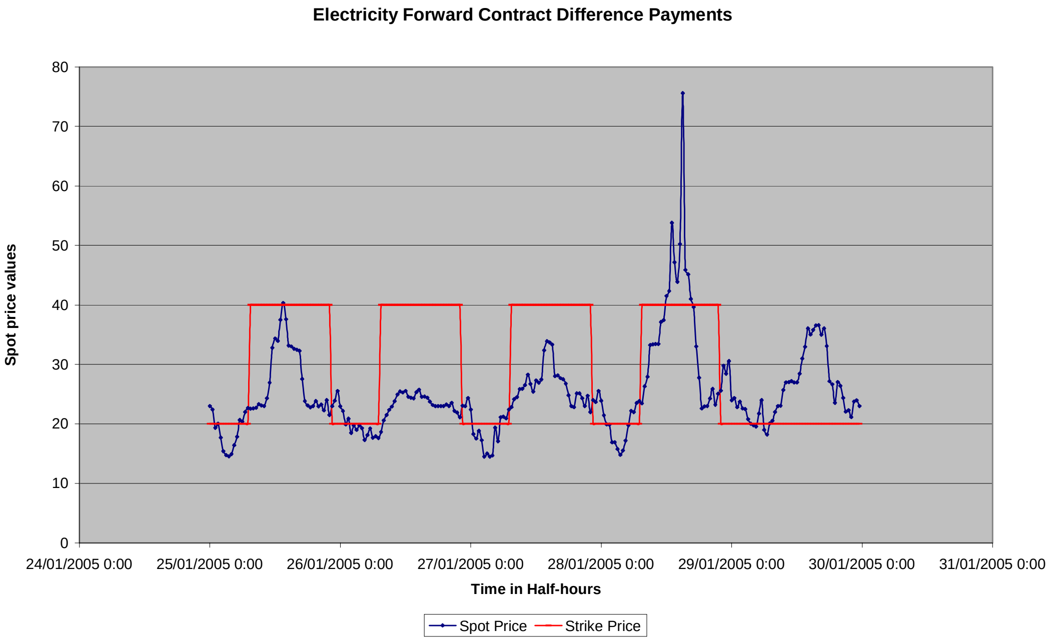
In the presence of optimal investment in tradeable assets at a given level of wealth (x) we are indifferent between π now and G in one year's time.

- If the utility function u is such that Arrow-Pratt risk aversion parameter $-\frac{u''(x)}{u'(x)}$ is independent of the states of the nature, then

$$\pi \approx \frac{1}{R}\left[\mathsf{E}[G] + \frac{1}{2}\frac{u''(W)}{u'(W)}\mathbf{var}[G] \right]$$

- Similar to the actuarial variance principal with loading factor of $\frac{u''(W)}{u'(W)}$

- Definition of Electricity forward contract



- Difference payments from a forward contract spanning n weeks is

$$\sum_{w=1}^{n} e^{-rw} \sum_{i=1}^{M_w} (S_i - F)L = \sum_{w=1}^{n} e^{-rw} M_w L [P_w - F]$$

where M_w is the number of half-hours contracted in week w, S_i is the pool spot price for the half-hour i, P_w is the average price for week w and F : the strike price of the contract ($/MWh)

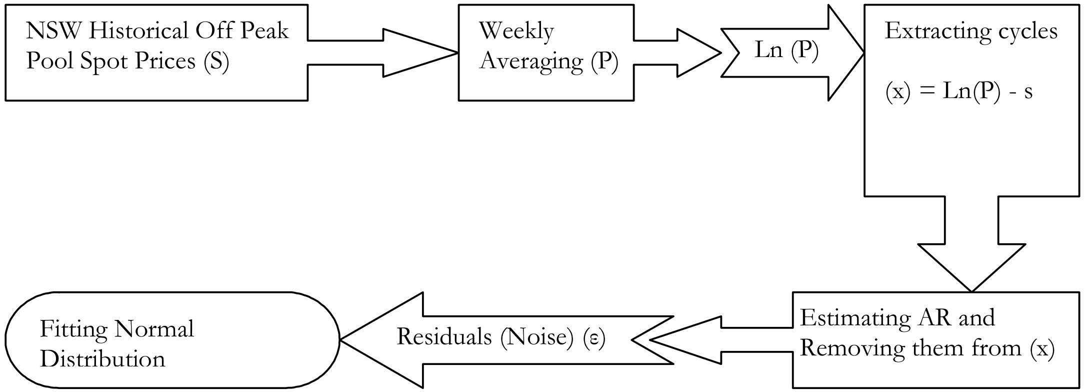
- Suppose that the contract spans 1 week. If S_t were known, then the fair forward price F could be chosen such that

$$F = \frac{1}{M_w} \sum_{t=1}^{M_w} S_t$$

- The usual approach in no-arbitrage models consists of putting

$$F = \frac{1}{M_w} \sum_{t=1}^{M_w} \mathsf{E}^Q[S_t]$$

- Electricity prices characteristics: seasonality, cycles, autocorrelation, by time partitions
- Modelling is done by Peak and offpeak



- After fitting a linear model to x, we obtain an $AR(3)$ model

$$\ln(P_t) = s_t + x_t$$

where $\{s_t\}$ is a deterministic seasonal component, and $\{x_t\}$ is modelled by an $AR(3)$ process

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3} + \sigma \varepsilon_t$$

- Electricity is a real asset so: $\pi = P_{t-1}$ and $G = \lambda P_t$ where $0 < \lambda < 1$ is a constant.

- $-\log(\lambda)$ can be thought of as a storage cost (of fuel) per megawatt hour per week. With exponential utility functions:

$$P_{t-1} = \frac{1}{R}\left[\mathsf{E}_{t-1}[\lambda P_t] + \frac{\gamma}{2} var_{t-1}[\lambda P_t] \right]$$

- The risk aversion parameter γ_{t-1} can be inferred from the above as follows

$$\gamma_{t-1} = \frac{-\mathsf{E}_{t-1}[\lambda P_t] + RP_{t-1}}{\frac{\lambda^2}{2} var_{t-1}[P_t]}$$

where $R = e^{r\Delta t}$, r is the risk-free rate of interest and $\Delta t = 1/52$ (1 week).

- The settlement amount for a forward contract spanning n weeks is

$$\sum_{w=1}^{n} e^{-rw} \sum_{i=1}^{M_w} (S_i - F)L = \sum_{w=1}^{n} e^{-rw} M_w L[P_w - F]$$

- where M_w is the number of half-hours contracted in week w, S_i is the pool spot price for the half-hour i, P_w is the average price for week w and F is the strike price.
- Let V_t^n be the present value of this contract (asking price), then V_t^n is the ask price of

$$\overline{L}_{t+1}(P_{t+1} - F) + V_{t+1}^{n-1} = Z_{t+1}$$

- So

$$V_t^n = \frac{1}{R}\left[\mathsf{E}_t[Z_{t+1}] + \frac{\gamma_t}{2} var_t[Z_{t+1}] \right]$$

where

$$\mathsf{E}_t[Z_{t+1}] = \overline{L}_{t+1}(\mathsf{E}_t[P_{t+1}] - F) + \mathsf{E}_t[V_{t+1}^{n-1}]$$

and

$$var_t[Z_{t+1}] = \overline{L}_{t+1}^2 var_t[P_{t+1}] + var_t[V_{t+1}^{n-1}] + 2\overline{L}_{t+1} cov_t(P_{t+1}, V_{t+1}^{n-1})$$

and

$$\gamma_t = \frac{RP_t - \lambda E_t[P_{t+1}]}{\frac{\lambda^2}{2} var_t[P_{t+1}]}$$

- We assume that the process of weekly average prices, $(P_t)_{t\geq 0}$ is such that

$$P_t = e^{s_t + x_t}$$

where $\{s_t\}$ is a deterministic seasonal component, and $\{x_t\}$ is modelled by an $AR(3)$ process

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3} + \sigma_t \varepsilon_t$$

- Lemma

$$\mathsf{E}_t[P_{t+1}^{\theta}] = M(t,\theta) P_t^{\theta\phi_1} P_{t-1}^{\theta\phi_2} P_{t-2}^{\theta\phi_3}$$

where

$$M(t,\theta) = \exp\left[\theta s_{t+1} - \phi_1\theta s_t - \phi_2\theta s_{t-1} - \phi_3\theta s_{t-2} + \frac{1}{2}\sigma_{t+1}^2\theta^2\right]$$

- and

$$var_t[P_{t+1}^{\theta}] = N(t,\theta) P_t^{2\theta\phi_1} P_{t-1}^{2\theta\phi_2} P_{t-2}^{2\theta\phi_3}$$

where

$$N(t,\theta) = \left[1 - e^{-\theta^2\sigma^2}\right] M(t,2\theta)$$

- For each time $t > 0$ and for any real numbers $\nu,\eta > 0,$ we have

$$cov_t(P_{t+1}^{\nu}, P_{t+1}^{\eta}) = M(t,\nu+\eta)(1 - e^{-\sigma^2\eta\nu}) P_t^{(\nu+\eta)\phi_1} P_{t-1}^{(\nu+\eta)\phi_2} P_{t-2}^{(\nu+\eta)\phi_3}$$

- The present value of the one week contract is

$$V_t^1 = \frac{1}{R}\left[\mathsf{E}_t[Z] + \frac{\gamma_t}{2} var_t[Z] \right]$$

where

$$Z = \overline{L}_{t+1}(P_{t+1} - F)$$

- This gives a starter value for V_t^n in the recursion formula.

- Therefore, the week-ahead forward price is given by

$$F_t(1) = \frac{1}{c_t^1}\left(a_t^1 P_t + b_t^1 M_t P_t^{\phi_1} P_{t-1}^{\phi_2} P_{t-2}^{\phi_3} \right)$$

where

$$\begin{cases} a_t^1 = \dfrac{\overline{L}_{t+1}^2}{\lambda^2} \\ b_t^1 = \dfrac{\overline{L}_{t+1}}{R} - \dfrac{\overline{L}_{t+1}^2}{\lambda R} \\ c_t^1 = \dfrac{\overline{L}_{t+1}}{R} \end{cases}$$

• By induction, we can prove that the forward price corresponding to the n-period contract is given by

$$F_t(n) = \frac{1}{c_t^n} \sum_k a_t^n(k) P_t^{\alpha_k^n} P_{t-1}^{\beta_k^n} P_{t-2}^{\gamma_k^n}$$

where

$$c_t^{n+1} = \frac{1}{R} \overline{L}_{t+1} + \frac{1}{R^2} \overline{L}_{t+2} + \ldots + \frac{1}{R^{n+1}} \overline{L}_{t+n+1}$$

and $a_t^n(k)$ α_k^n, β_k^n and γ_k^n are deterministic coefficient, computed in a recursive fashion.

- Calibration of the forward prices to market data

- Markov chains probabilities approximation for efficient calculation of $a_t^n(k)$ α_k^n, β_k^n and γ_k^n