On the Impact of Heavy-Tailed Returns to Popular Risk Measures: Evidence from Global Indices

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Outline

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- Background on Risk Measures
- Heavy-Tailed Nature of Returns and Drawdowns
- Stable Returns: Effect on Risk Measures
- Stable factors and Risk Measures
- Conclusions

Motivation

- Drawdown: Chekhlov et.al.,(2000); Magdon-Ismail et.al.(2004);
  Acar and James(1997); Johansen and Sornette(2001);
  Burghardt et.al.(2003); etc.
- real financial data present asymmetries and fat tails.
- Stable distributions in many areas of finance and insurance,
  Mittnik and Rachev (2000), Schwatz (2002),
  Harmantzis et.al.(2005), etc.

Objective

- explore the properties of Drawdown-type measures.
- use α-Stable Paretian assumption to estimate popular Drawdown-type and VaR-type risk measures, by real data.
- analyze the impact of key factors on the performance of risk measures

VaR-Type Risk Measures

- VaR: $P(X > \mathrm{VaR}_\alpha) \leq 1 - \alpha$
- CVaR: $CVaR_\alpha = E[X|X \geq VaR_\alpha(X)]$, Artzner et al.(1997)

X: the negative P& L of a risky financial position.

α: confidence level

Property of ES:

- CVaR considers size of loss beyond the VaR level.
- CVaR is more conservative than VaR.
- CVaR is coherent.

Drawdown-Type Risk Measures

- $DD(t) = \max_{0 \le \tau \le t} X(\tau) - X(t)$.

  where $X(t)$ is the cumulative return.

- $CDaR_\alpha = \frac{1}{(1-\alpha)T} \int_{t \in [0,T], DD(t) \ge \zeta(\alpha)} DD(t)dt$.

  where $\zeta(\alpha)$ is DaR, the threshold such that $(1-\alpha) * 100\%$ of Drawdowns exceed it.

  Two special cases of CDaR: MDD($\alpha = 1$) and AvDD($\alpha = 0$).

- $MDD = \max_{0 \le t \le T} DD(t)$.

- $AvDD = \frac{1}{T} \int_0^T DD(t)dt$.

Stable Paretian Model

The random variable X is stable distributed with parameters α, β, μ and σ ($X \sim S_\alpha(\mu, \sigma, \beta)$) if

$$\log Ee^{itX} = \begin{cases} -\sigma^\alpha |t|^\alpha (1 - i\beta \,\mathrm{sgn}(t) \tan \frac{\pi\alpha}{2}) + i\mu t & (\alpha \neq 1) \\ -\sigma |t| (1 + i\beta \frac{2}{\pi} \,\mathrm{sgn}(t) \log |t|) + i\mu t & (\alpha = 1) \end{cases}$$

- $\alpha \in (0, 2]$: index of stability
- $\beta \in [-1, 1]$: skewness parameter
- $\sigma \in \mathbb{R}^+$: scale parameter
- $\mu \in \mathbb{R}$: location parameter

If $\alpha = 2, \beta = 0$, Stable Paretian distribution reduces to Gaussian distribution.

Stable Paretian Risk Measures

Given the observed returns r_i,

- Parameter estimation by MLE,
  Mittnik, Rachev, Doganoglu, and Chenyao(1999)
  Nolan(2001)

- Monte Carlo Simulations to compute VaR and ES,
  Chamber, Mallows and Stuck(1976)
  Compute stable risk measures as the definitions

Table 1: Parameter Estimation of α-Stable Distribution for Logarithm Daily Returns of Global Indices 01/03/1995-12/31/2004

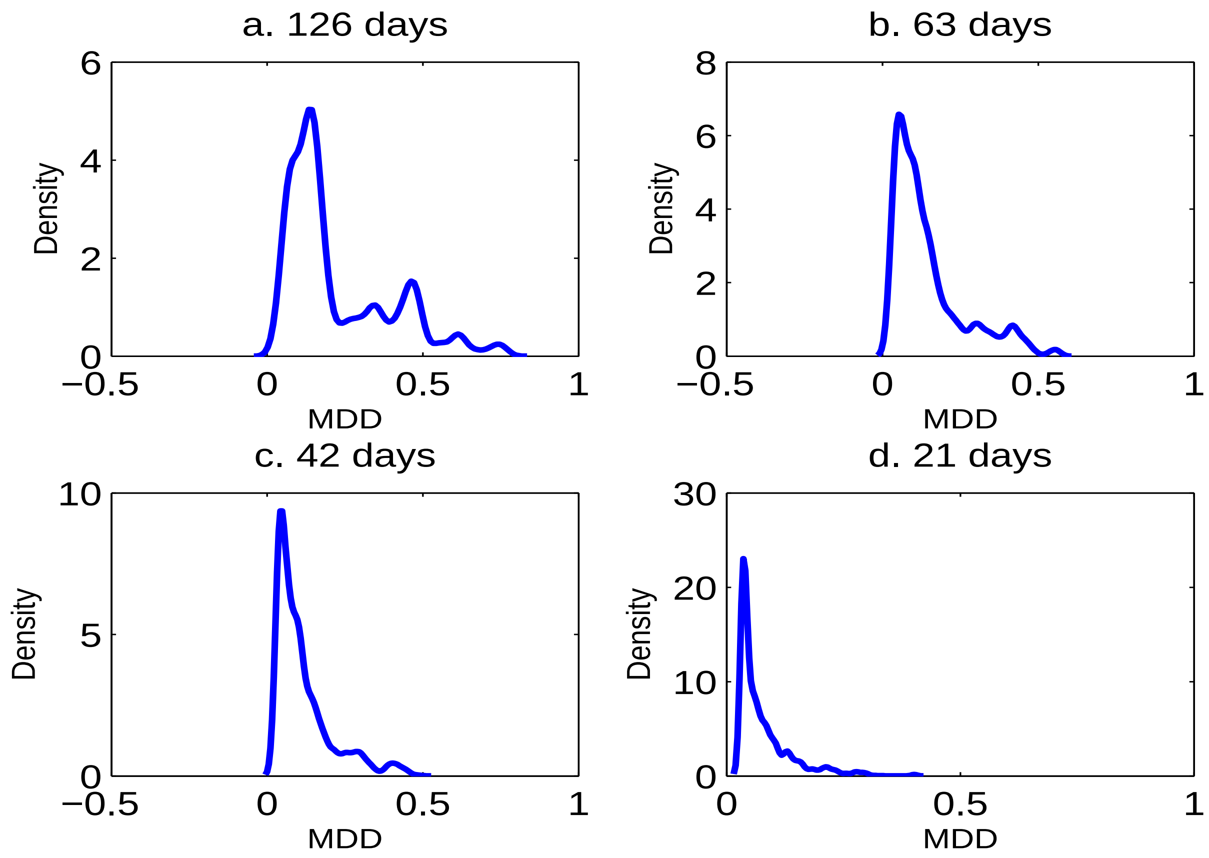
Series	Days	Sd	Mean	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\mu}$
S&P500	2516	0.012	3.86E-04	1.565	-0.108	6.28E-03	7.24E-04
NASDAQ	2518	0.018	4.26E-04	1.494	-0.205	9.34E-03	2.03E-03
NIKKEI	2461	0.015	-2.19E-04	1.679	-0.035	8.89E-03	-1.72E-04
FTSE	2525	0.011	1.79E-04	1.539	-0.068	6.15E-03	5.64E-04
DAX	2522	0.016	2.84E-04	1.550	-0.139	8.68E-03	1.05E-03
CAC	2525	0.015	2.80E-04	1.624	-0.169	8.32E-03	8.57E-04
HSI	2470	0.017	2.41E-04	1.532	-0.029	8.74E-03	2.21E-04
AORD	2534	0.008	2.98E-04	1.693	-0.027	4.43E-03	4.00E-04

α-Stable Parameter Estimation for NASDAQ MDD with Different Moving Tracking Time T

Series	T	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\mu}$
NASDAQ	126	2.000	-	0.125	0.152
	63	1.402	1.000	0.058	0.107
	42	1.572	1.000	0.048	0.084
	21	1.639	1.000	0.039	0.055

- Both $\hat{\alpha}$ and $\hat{\mu}$ decrease when the tracking time decreases.
- Besides cases where $\hat{\alpha}$ equals 2, $\hat{\beta}$ values are equal to 1 for all tracking time scenarios and data sets without exception.

Kernel Densities of MDD for NASDAQ.



Modeling Stable Returns: Effect on Risk Measures (1)

- Target: “Stable risk measures”
  Stable parameters; generating random variables; calculating risk measures; repeating 10,000 times; computing the means.
- Benchmark: “Normal risk measures”
  means and standard deviations; Normally generating random variables; calculating risk measures; repeating 10000 times; computing the means.
- Comparison: “Empirical risk measures”
- compare accuracy of Stable and Normal risk measures
  $Bias = (ModelValue - EmpiricalValue)/EmpiricalValue.$

Modeling Stable Returns: Effect on Risk Measures (2)

- risk averse investors: positive smaller *Biases*.
- Stable modeling of returns always overestimates risk measures.
  Normal modeling underestimates risk (except VaR).
- Absolute Value of *Biases*:
  Stable VaR is less than Normal VaR
  Stable CVaR is larger than Normal CVaR
  Drawdown-type measures are not consistent for all data series.
- Stable assumption produces more accurate and conservative risk measures

Table 2: Biases of Stable Risk Measure Estimates

Series	MDD	CDaR95	DaR95	AvDD	CVaR95	VaR95
S&P 500	41.3%	13.0%	-18.5%	-6.7%	96.2%	3.4%
NASDAQ	17.8%	14.1%	6.4%	-21.2%	63.5%	6.2%
NIKKEI	58.3%	60.1%	55.4%	63.7%	42.2%	2.1%
FTSE	12.7%	24.8%	27.5%	58.8%	58.8%	0.6%
DAX	42.5%	59.1%	65.3%	93.5%	42.3%	3.6%
CAC	13.7%	20.0%	19.6%	29.1%	45.9%	3.6%
HSI	62.6%	68.8%	64.2%	93.4%	38.2%	1.2%
AORD	59.6%	88.6%	93.7%	114.5%	47.9%	0.8%

Table 3: Biases of Gaussian (Normal) Risk Measure Estimates

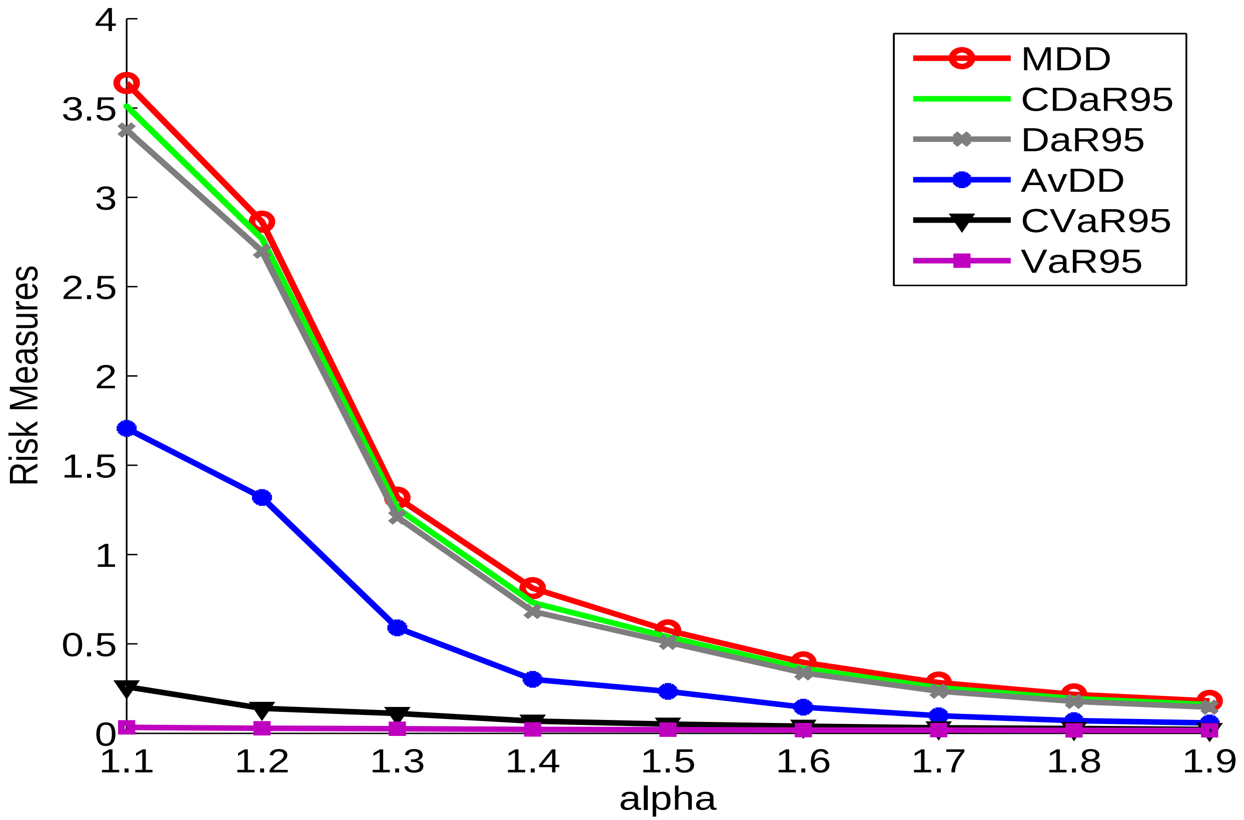
Series	MDD	CDaR95	DaR95	AvDD	CVaR95	VaR95
S&P 500	-35.0%	-34.8%	-35.8%	-15.0%	-8.7%	7.3%
NASDAQ	-47.4%	-49.1%	-51.4%	-40.8%	-8.4%	10.9%
NIKKEI	7.9%	8.2%	8.0%	23.9%	-7.3%	4.3%
FTSE	-27.8%	-21.9%	-21.2%	8.1%	-8.6%	3.9%
DAX	-42.6%	-38.8%	-37.6%	-15.3%	-7.0%	7.7%
CAC	-37.4%	-34.2%	-34.3%	-11.3%	-7.3%	8.0%
HSI	-6.1%	1.5%	1.8%	6.2%	-9.3%	11.6%
AORD	8.0%	32.0%	37.6%	73.7%	-1.8%	7.5%

Stable factors and Risk Measures

- mean returns
- standard deviation of returns
- excess Kurtosis: α
- negative Skewness: β
- length of tracking time: T

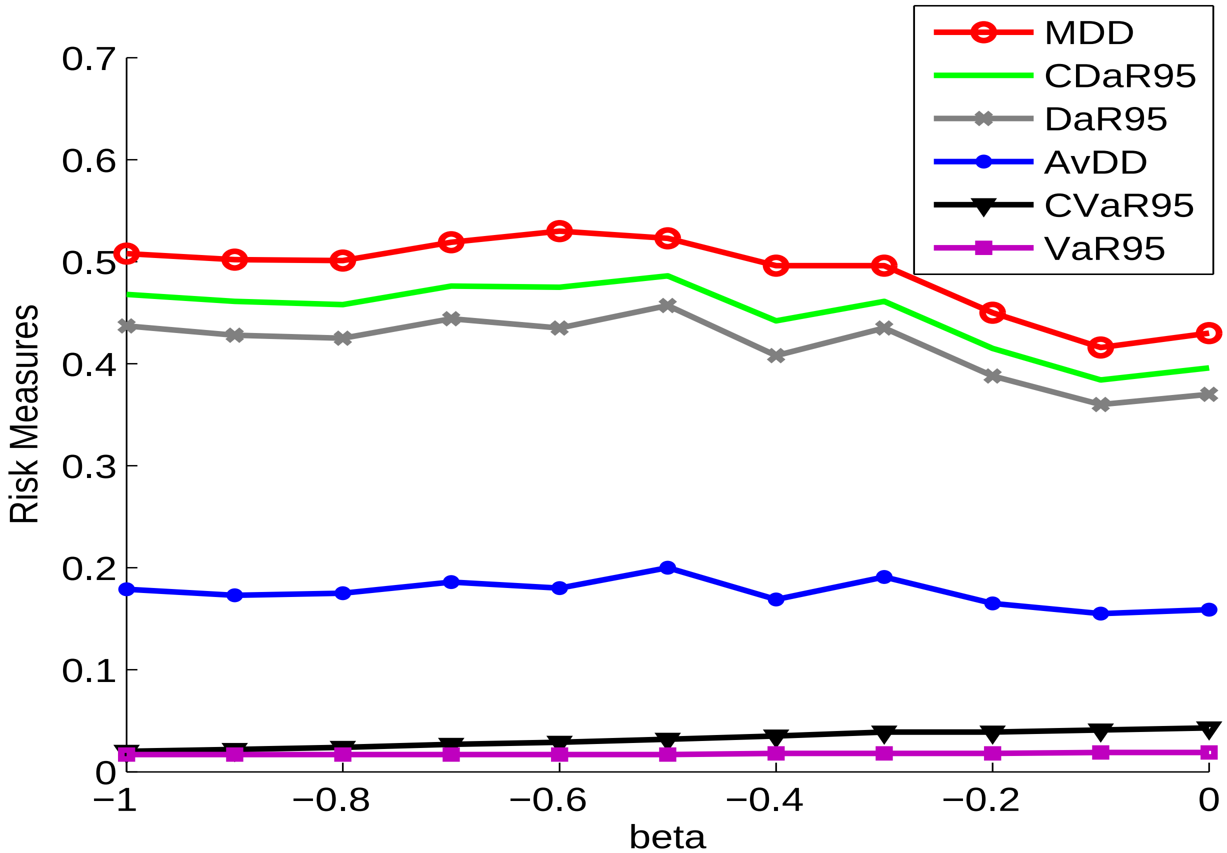
Thickness of Return Tails on Risk Measures

- The greater the excess kurtosis,
  the larger the Drawdown-type risk measures.
- Higher kurtosis induces higher CVaR,
  but has no apparent effect on VaR.


Risk Measures
4
3.5
3
2.5
2
1.5
1
0.5
0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
alpha
MDD
CDaR95
DaR95
AvDD
CVaR95
VaR95

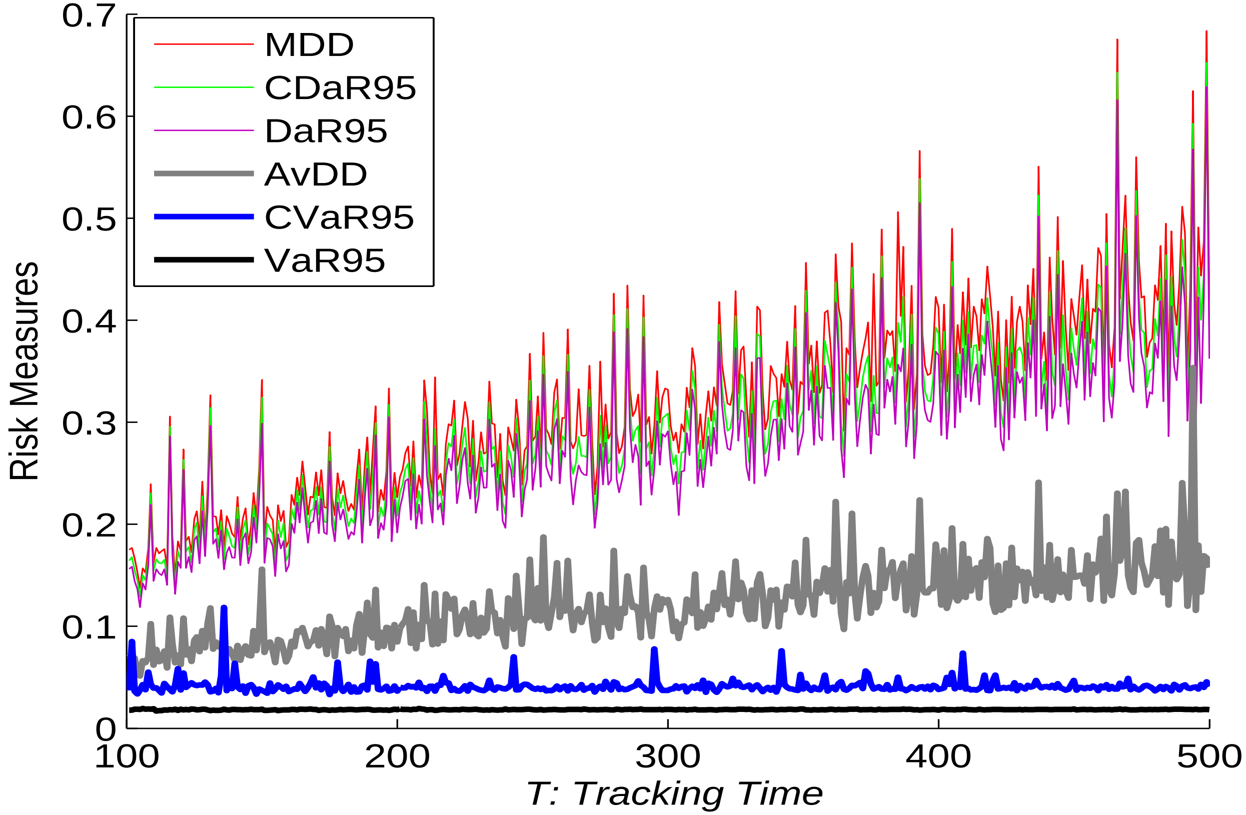
Skewness of Returns on Risk Measures

- The skewness does not appear to be an important factor for Drawdown-type risk measures.
- Skewness does affect CVaR, which is getting larger as the distribution becomes more symmetric.
- VaR does not change with the skewness level.


MDD
CDaR95
DaR95
AvDD
CVaR95
VaR95
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Risk Measures
−1
−0.8
−0.6
−0.4
−0.2
0
beta

Effect of Tracking Time on Risk Measures

- The number of generated points of Monte Carlo simulations stands for the length of the tracking time in days.
- The wider the tracking time, the larger the risk measures, except VaR.



MDD
CDaR95
DaR95
AvDD
CVaR95
VaR95
Risk Measures
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
T: Tracking Time

Conclusions

- Stable assumption produces more accurate and conservative risk measures.
- Drawdown-type and VaR-type risk measures do not necessarily agree on the magnitude/risk levels they communicate to investors.
- Drawdown and VaR-type risk measures are complements not supplements.
- Extensions

  Drawdown measures for hedge funds

  More Applications of Stable model