Statistical Inference for Dependent Credit Events

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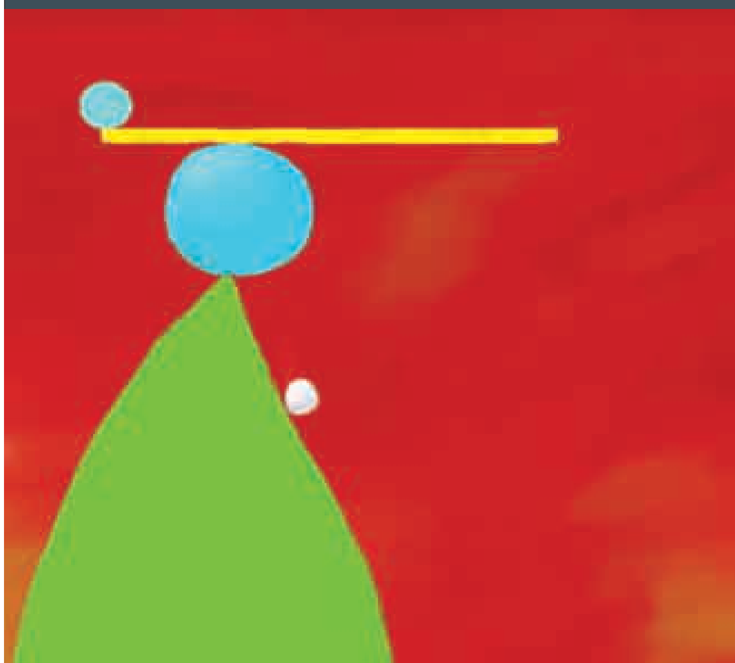
December 2005

QUANTITATIVE RISK MANAGEMENT



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Concepts, Techniques and Tools

PRINCETON SERIES IN FINANCE

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1.1 Motivation: Dependent Defaults

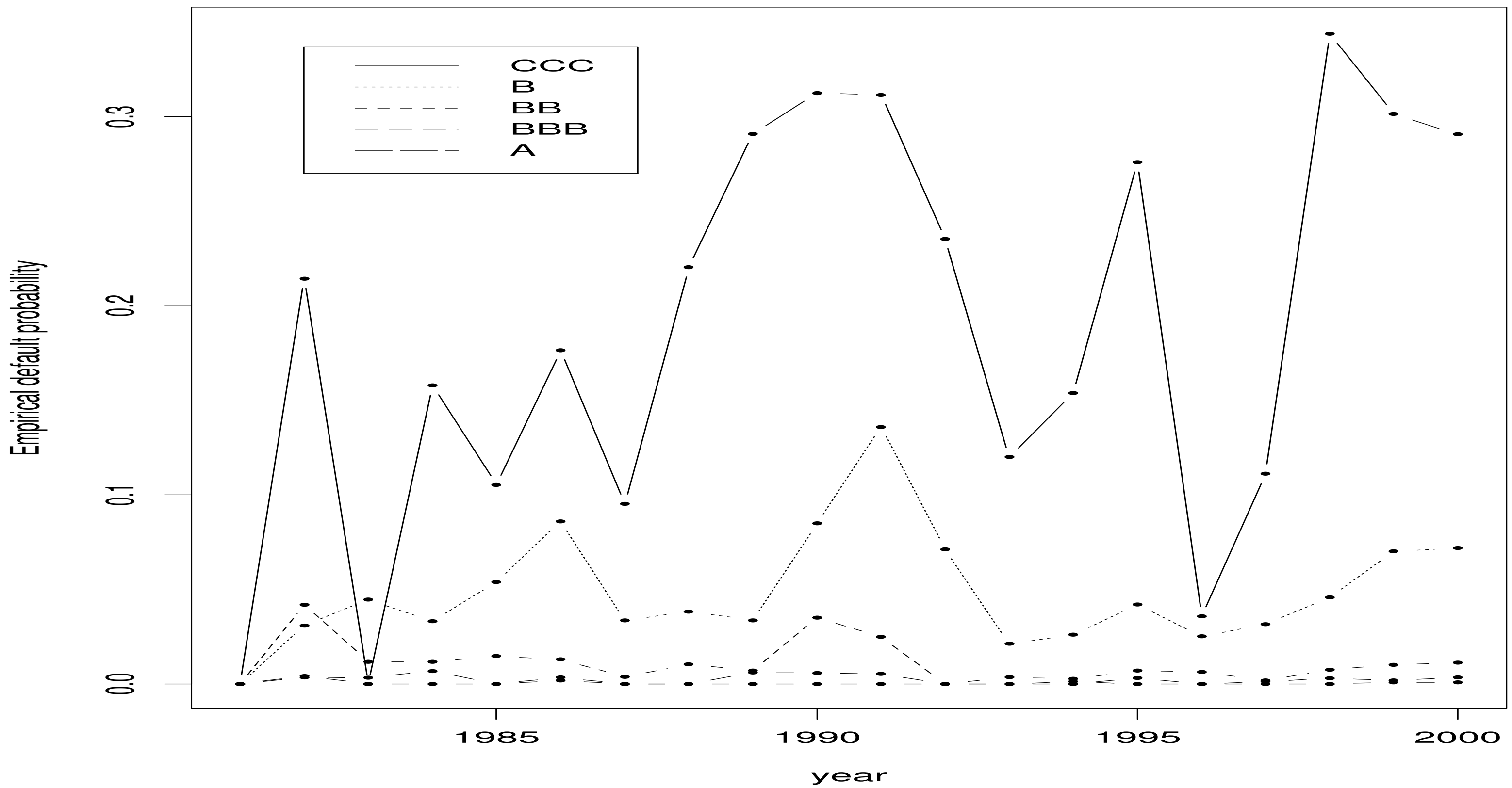
Dependence between defaults is key issue in credit risk management.

- In large balanced loan portfolios main risk is occurrence of many joint defaults – this might be termed extreme credit risk.
- Dependence between default critically affects performance of many basket credit derivatives

Sources for dependence between defaults

- Dependence caused by common factors (eg. interest rates and changes in economic growth) affecting all companies
- Default of company A may have direct impact on default probability of company B and vice versa because of direct business relations, a phenomenon known as counterparty risk or contagion.

Empirical Evidence



Standard and Poor's default data from 1980 to 2000 show clear evidence of cycles; we expect within-year and between-year dependence.

1.2 Bernoulli Mixture Models

These provide a way of capturing the dependence among Bernoulli events (i.e. defaults/non-defaults) occurring in a fixed time period.

Notation. We consider m credit risks and a vector of default indicators $\mathbf{Y} = (Y_1, \ldots, Y_m)'$, where $Y_i = 1$ if counterparty i defaults in the time period and $Y_i = 0$ otherwise.

Definition. (Bernoulli mixture model). Given some $p < m$ and a p-dimensional random vector $\mathbf{\Psi} = (\Psi_1, \ldots, \Psi_p)'$, the default indicator vector $\mathbf{Y}$ follows a Bernoulli mixture model with factor vector $\mathbf{\Psi}$ if there are functions $p_i : \mathbb{R}^p \to (0, 1)$, such that conditional on $\mathbf{\Psi}$ the components of $\mathbf{Y}$ are independent Bernoulli rvs with $P(Y_i = 1 \mid \mathbf{\Psi} = \boldsymbol{\psi}) = p_i(\boldsymbol{\psi})$.

Distribution of Defaults

For $\mathbf{y} = (y_1, \dots, y_m)'$ in $\{0,1\}^m$ we get

$$P(\mathbf{Y} = \mathbf{y} \mid \mathbf{\Psi} = \boldsymbol{\psi}) = \prod_{i=1}^{m} p_i(\boldsymbol{\psi})^{y_i}(1 - p_i(\boldsymbol{\psi}))^{1-y_i},$$

and the unconditional distribution is given by

$$f(\mathbf{y}) = P(\mathbf{Y} = \mathbf{y}) = \int_{\mathbb{R}^p} \prod_{i=1}^{m} p_i(\boldsymbol{\psi})^{y_i}(1 - p_i(\boldsymbol{\psi}))^{1-y_i} g(\boldsymbol{\psi}) \mathrm{d}\boldsymbol{\psi},$$

where $g(\boldsymbol{\psi})$ is the probability density of the factors.

By adding exposures and assumptions about losses given default we complete the specification of a one-period model.

1.3 The KMV/CreditMetrics Model

These industry models belong to the class of structural or firm-value models descending from Merton's influential credit risk model. [Merton, 1974]

They can be shown to be equivalent to Bernoulli mixture models with conditional default probabilities

$$p_i(\boldsymbol{\psi}) = \Phi\left(\frac{\Phi^{-1}(\bar{p}_i) - \mathbf{z}_i'\boldsymbol{\psi}}{\sqrt{1-\beta_i}}\right),$$

where $\bar{p}_i$ is the unconditional default probability of company i, $\boldsymbol{\Psi} \sim N_p(\mathbf{0}, \Omega)$ is a normally distributed factor vector, $\mathbf{z}_i$ are factor loadings and $\beta_i = \mathbf{z}'\Omega\mathbf{z}_i$ is the systematic risk component of company i.

Special Cases

One-Factor Model

$$p_i(\psi) = \Phi\left(\frac{\Phi^{-1}(\bar{p}_i) - \sqrt{\beta_i}\psi}{\sqrt{1-\beta_i}}\right),$$

where Ψ is a standard normally distributed factor.

Homogeneous Correlation Model

$$p_i(\psi) = \Phi\left(\frac{\Phi^{-1}(\bar{p}_i) - \sqrt{\rho}\psi}{\sqrt{1-\rho}}\right), \qquad (1)$$

where ρ is known as the asset correlation.

These are influential simple models, which have played a role in Basel II capital formulas.

1.4 Industry Approaches to Model Calibration

Industry models generally separate the problems of estimating (i) default probabilities and (ii) model parameters describing the dependence of defaults.

1. Default probabilities are usually estimated by a historical default rate for "similar" companies, where the similarity metric may be based on ratings (CreditMetrics) or a proprietary measure like distance-to-default (KMV).

2. Dependence is usually described by a macro-economic or fundamental factor model. Parameters of factor models are often simply "assigned" by economic argument or derived from factor analyses of proxy variables (e.g. equity returns for asset value returns in the firm-value models).

Ad Hoc Calibration and Model Risk

The ad hoc nature of some of the attempts to model dependence in particular raises the issue of model risk.

For example, in KMV/CreditMetrics, how confident are we that we can correctly determine the size of the systematic component of risk (determined by the common factors and loadings)?

Misspecification of the asset correlation parameter ρ in (1) can have a drastic impact on any calculations that are made with this model, for example determination of VaR.

This talk is about using historical default and migration data to estimate model parameters by proper statistical means.

2. Modelling Dependent Defaults

1. Bernoulli Mixture Models as GLMMs

2. Estimation of Models

3. Empirical Analysis of S&P Default Data

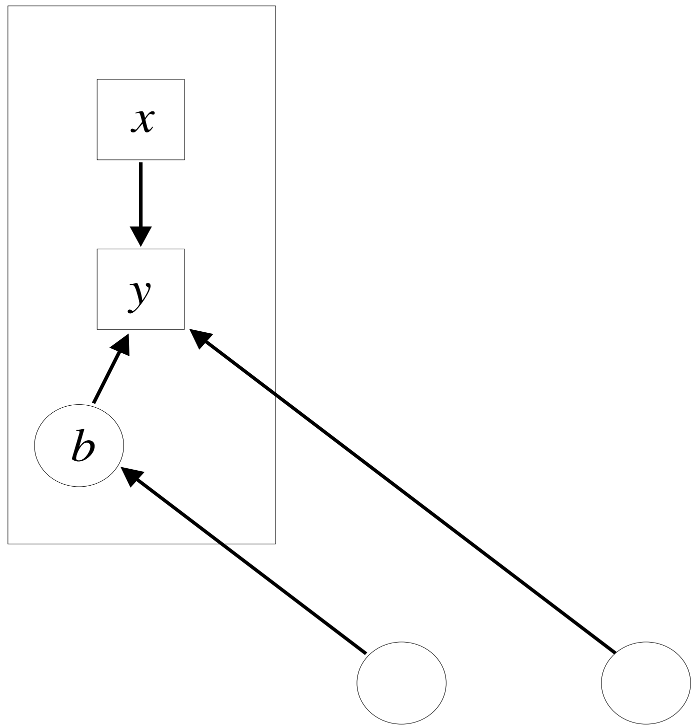
2.1 Bernoulli Mixture Models and GLMMs

We consider Bernoulli mixture models that have the structure of generalized linear mixed models (GLMMs). Conditional on factors $\mathbf{\Psi}$ (known in GLMMs as random effects) we assume a one-period model of form:

$$p_i(\boldsymbol{\psi}) = P(Y_i = 1 \mid \mathbf{\Psi} = \boldsymbol{\psi}) = g\left(\mathbf{x}_i'\boldsymbol{\beta} + \mathbf{z}_i'\boldsymbol{\psi}\right), \tag{2}$$

- where $g(\cdot)$ is a link function, typically a mapping from $\mathbb{R}$ to $(0, 1)$ like a distribution function (e.g. $g = \Phi$);
- $\mathbf{x}_i$ and $\mathbf{z}_i$ are explanatory variables (covariates) for ith company, such as indicators for rating category or sector, or firm-specific information from balance sheet;
- $\boldsymbol{\beta}$ are unknown parameters (generally including an intercept).

GLMM as DAG (Directed Acyclic Graph)



N.B. the factors $\mathbf{\Psi}$ are represented here by b and θ are hyperparameters of the distribution of $\mathbf{\Psi}$.

A Multi-Period Model

Given factors $\mathbf{\Psi}_t$ in time period t we assume that individual default indicators $Y_{t1}, \ldots, Y_{tm_t}$ are conditionally independent with

$$Y_{ti} \mid \mathbf{\Psi}_t = \boldsymbol{\psi}_t \sim \mathrm{Be}(p_{ti}(\boldsymbol{\psi}_t)), \quad i = 1, \ldots, m_t, \quad \text{where}$$
$$p_{ti}(\boldsymbol{\psi}_t) = g(\mu_{\kappa(t,i)} - \mathbf{x}'_{ti}\boldsymbol{\beta} - \mathbf{z}'_{ti}\boldsymbol{\psi}_t). \qquad (3)$$

Note some slight notational changes:

- $\kappa(t, i)$ returns the credit rating of company i,
- $\mu_1, \ldots, \mu_K$ are unknown intercepts.

The Asset Value Interpretation

Let $\epsilon_{t1}, \ldots, \epsilon_{tm_t}$ be iid rvs with distribution function g, which are also independent of $\mathbf{\Psi}_t$. Set $V_{ti} := \mathbf{x}'_{ti}\boldsymbol{\beta} + \mathbf{z}'_{ti}\mathbf{\Psi}_t + \epsilon_{ti}$ for $i = 1, \ldots, m_t$.

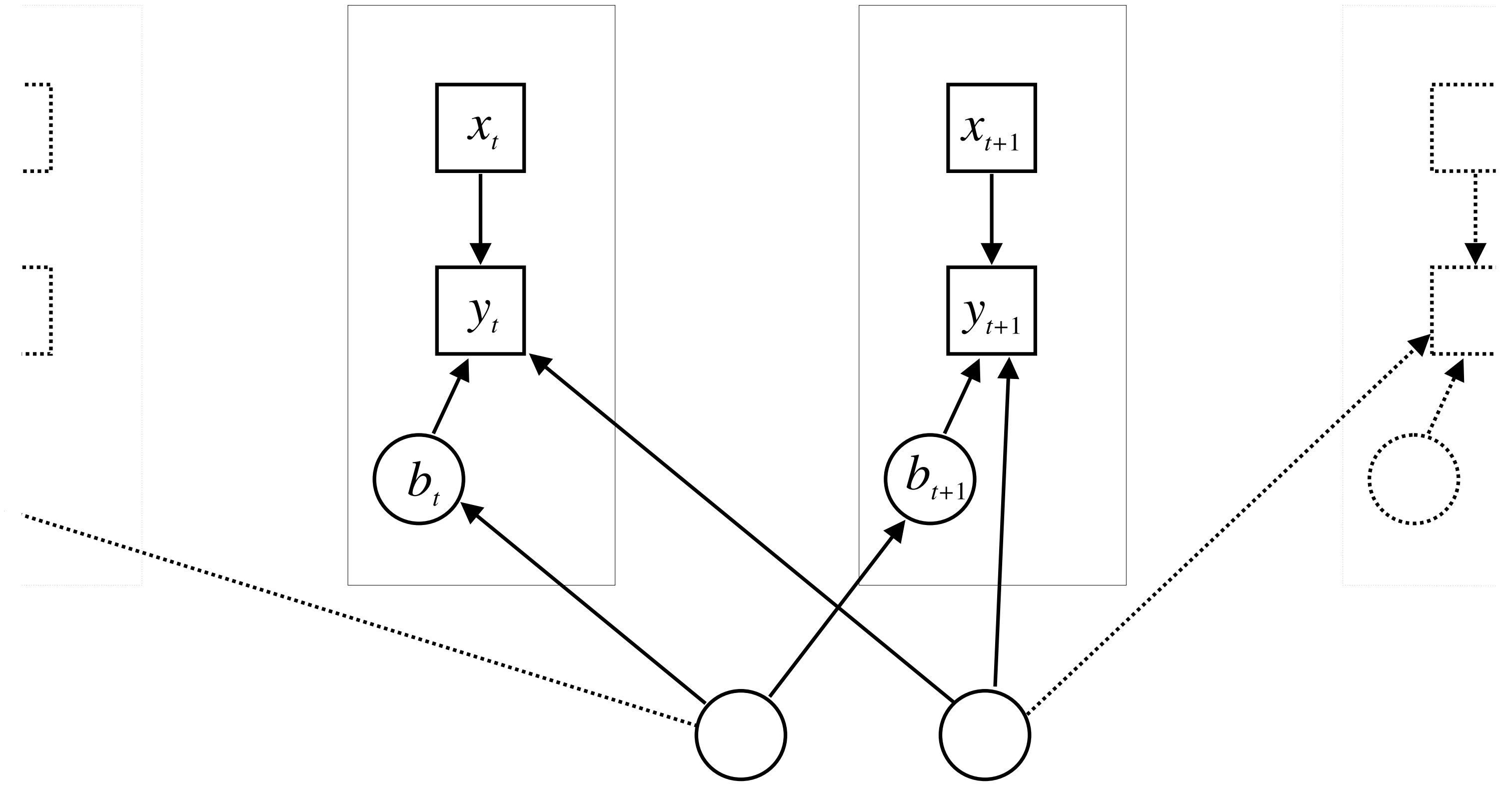
Observe that (3) corresponds to a model in which company i in period t defaults iff $V_{ti} \leq \mu_{\kappa(t,i)}$. The rv V_{ti} can be interpreted as the asset value and $\mu_{\kappa(t,i)}$ as the critical liability level.

The implied asset correlation of firms i and j in period t is

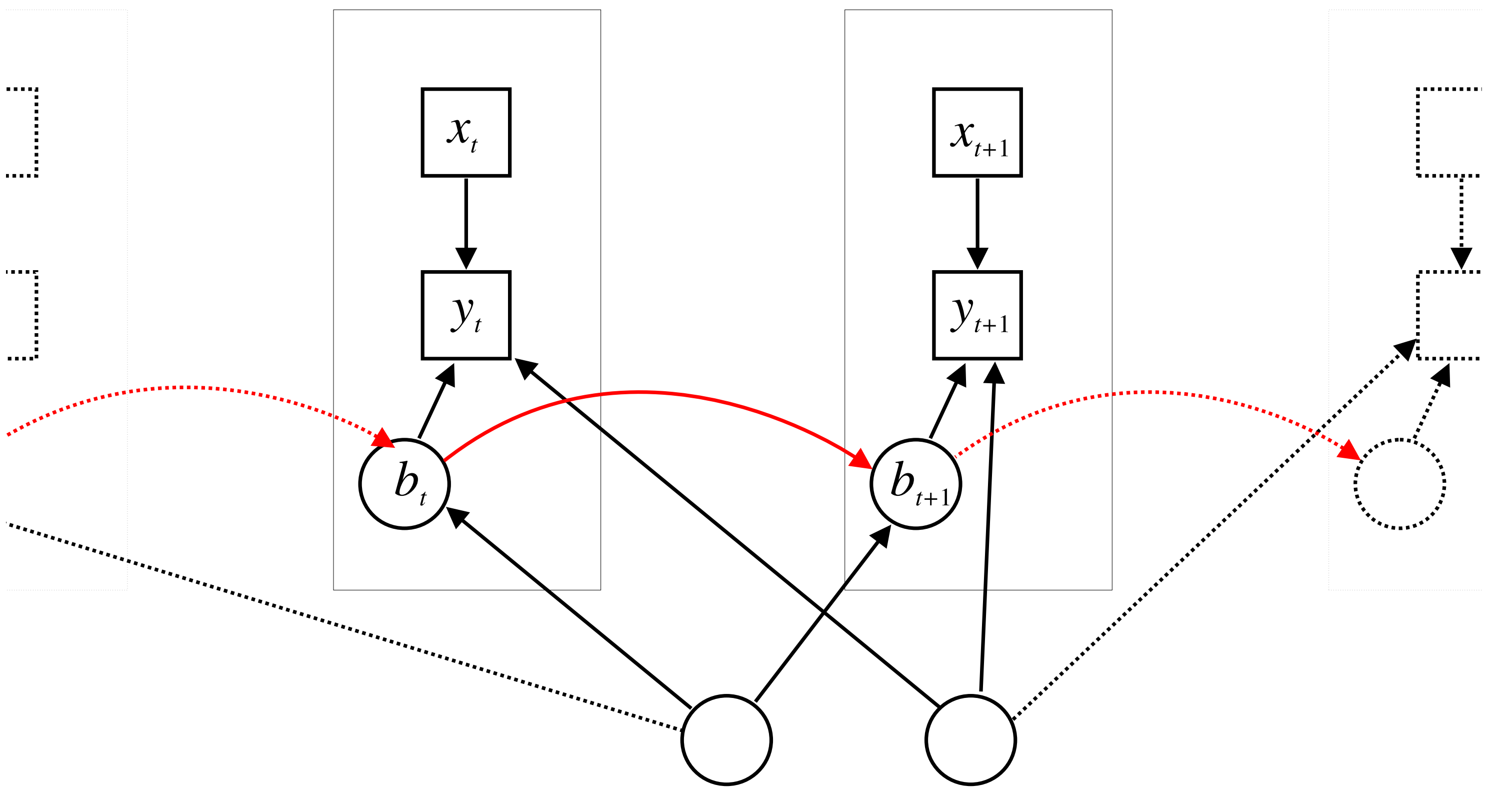
$$\rho(V_{ti}, V_{tj}) = \frac{\mathrm{cov}(\mathbf{z}'_{ti}\mathbf{\Psi}_t, \mathbf{z}'_{tj}\mathbf{\Psi}_t)}{\sqrt{\mathrm{var}(\mathbf{z}'_{ti}\mathbf{\Psi}_t) + w^2}\sqrt{\mathrm{var}(\mathbf{z}'_{tj}\mathbf{\Psi}_t) + w^2}}$$

where $w^2 := \mathrm{var}(\epsilon_{ti})$. Probit link: $w^2 = 1$. Logit link: $w^2 = \pi^2/3$.

Multi-Period Model



Multi-Period Model With Serial Dependence



Serial Dependence

Serial dependence in default probabilities can be modelled using correlated latent factors $\mathbf{\Psi}_1, \ldots, \mathbf{\Psi}_T$. We assume that

1. conditional on $(\mathbf{\Psi}_t)_{t=1}^T$ the default indicator vectors $\mathbf{Y}_1, \ldots, \mathbf{Y}_T$ are independent. Moreover, $\mathbf{Y}_t$ depends on $\mathbf{\Psi}_t$ only.

2. the latent factors $(\mathbf{\Psi}_t)_{t=1}^T$ form a Markov chain.

Remark: The above assumptions define a state space model (hidden Markov model) for the sequence $\mathbf{Y}_1, \ldots, \mathbf{Y}_T$.

2.2 Estimation of Models

IID random effects

The unconditional mass function of $\mathbf{Y}_t = (Y_{t1}, \ldots, Y_{tm_t})'$ is

$$f(\mathbf{y}_t \mid \boldsymbol{\beta}, \boldsymbol{\theta}) = \int_{\mathbb{R}^p} \left(\prod_{i=1}^{m_t} P(Y_{ti} = y_{ti} \mid \boldsymbol{\Psi}_t = \boldsymbol{\psi}, \boldsymbol{\beta}) \right) g(\boldsymbol{\psi} \mid \boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{\psi}$$

where $p = \text{dim}(\boldsymbol{\psi})$ and $g(\boldsymbol{\psi} \mid \boldsymbol{\theta})$ is the density of $\boldsymbol{\Psi}_t$. The likelihood function with $\boldsymbol{\Psi}_1, \ldots, \boldsymbol{\Psi}_T$ independent is

$$L(\boldsymbol{\beta}, \boldsymbol{\theta} \mid \text{observed data}) = \prod_{t=1}^{T} f(\mathbf{y}_t \mid \boldsymbol{\beta}, \boldsymbol{\theta}). \tag{4}$$

There is no between-period dependence.

Dependent Random Effects

Let $\boldsymbol{\Psi}_1, \ldots, \boldsymbol{\Psi}_T$ have joint density $g(\boldsymbol{\psi}_1, \ldots, \boldsymbol{\psi}_T \mid \boldsymbol{\theta})$. The likelihood function $L(\boldsymbol{\beta}, \boldsymbol{\theta} \mid \text{observed data})$ now takes the form

$$\int \cdots \int \prod_{t=1}^{T} \prod_{i=1}^{m_t} P(Y_{ti} = y_{ti} \mid \boldsymbol{\Psi}_t, \boldsymbol{\beta}) g(\boldsymbol{\psi}_1, \ldots, \boldsymbol{\psi}_T \mid \boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{\psi}_1 \cdots \mathrm{d}\boldsymbol{\psi}_T.$$

To evaluate this expression we have an integral over $\mathbb{R}^{T \times p}$, which makes standard maximum likelihood difficult.

Bayesian Inference

We distinguish between observed quantities $D := (\mathbf{x}_t, \mathbf{z}_t, \mathbf{y}_t)_{t=1}^T$ and unobserved quantities $\vartheta := (\boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{\psi}_1, \ldots, \boldsymbol{\psi}_T)$.

The prior distribution $p(\vartheta)$ expresses a state of knowledge (or ignorance) about the unobserved elements ϑ before the data D are obtained.

Inference in our model is based on the posterior distribution $p(\vartheta \mid D)$

$$p(\vartheta \mid D) = \frac{p(D \mid \vartheta)p(\vartheta)}{p(D)} = \frac{p(D \mid \vartheta)p(\vartheta)}{\int p(D \mid \vartheta)p(\vartheta)\,\mathrm{d}\vartheta}.$$

Problem: finding $p(\vartheta \mid D)$!

Markov Chain Monte Carlo (MCMC)

Assume we want to simulate from a (multivariate) distribution $p(\mathbf{x})$.

Idea: Construct an ergodic Markov chain with p as its stationary distribution. Regard a sample of the Markov chain (possibly after a certain burn-in) as a sample from p. Constructing such a Markov chain turns out to be surprisingly simple:

- Metropolis-Hastings algorithm
- Gibbs sampler (special case)

MCMC can be used to simulate $p(\vartheta \mid D)$ even in complex cases. [Robert and Casella, 1999, Clayton, 1996]

Advantages of MCMC

- calibration of complex models with multivariate, serially correlated latent factors; implementation straight-forward; simulation fast;
- point estimates, standard errors and (joint) confidence sets of parameters ϑ are inherent in the output;
- inference about derived model parameters (e.g. default correlations, implied asset correlations) as easy as for primary parameters;
- posterior path of latent factors $(\mathbf{\Psi}_t)$ can be compared with other macro-economic variables;
- prior information about parameters governing portfolio dependence can be entered in the analysis.

2.3 Empirical Analysis of S&P Default Data

Homogeneous portfolio: $\mathbf{x}_{ti} = \mathbf{x}_t$ for all companies.
Observed data collected on a six-month basis:

$M_{tk} := \sum_{i:\kappa(t,i)=k} Y_{ti}$	(number of *defaults* for rating class k),
$m_{tk} := \#\{i : \kappa(t,i) = k\}$	(number of *companies* for rating class k),
$\mathbf{M}_t := (M_{t1}, \ldots, M_{tK})'$	(vector of default count variables).

We fit several models to S&P default data (rating classes CCC, B, BB, BBB, A) by Gibbs sampling with non-informative priors. The sequence (Ψ_t) of scalar latent factors satisfies an AR(1) process:

$$\Psi_t = \alpha \Psi_{t-1} + \phi \epsilon_t, \qquad b_0 = \phi \epsilon_0 / \sqrt{1 - \alpha^2},$$

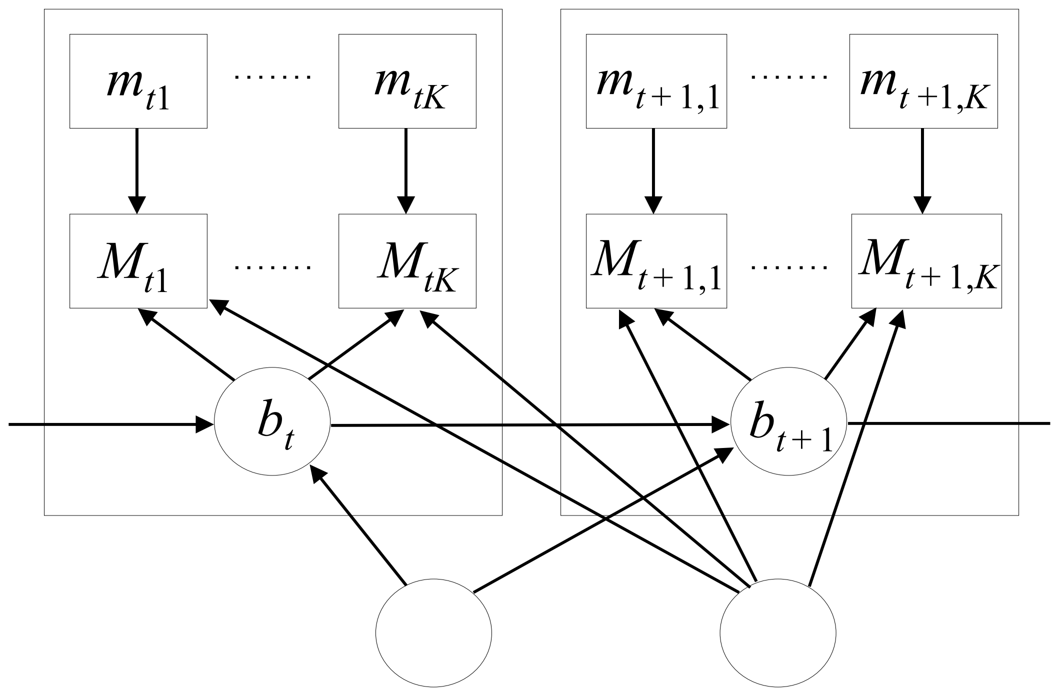
where $\epsilon_0, \epsilon_1, \ldots$ are iid $N(0,1)$.

Empirical Analysis

Given $(\Psi_t)_{t=1}^{T}$, we assume $\mathbf{M}_1, \ldots, \mathbf{M}_T$ conditionally independent.

Model 1: $\quad M_{tk} \mid \Psi_t = \psi_t \sim B(m_{tk},\, g(\mu_k - \psi_t)),$

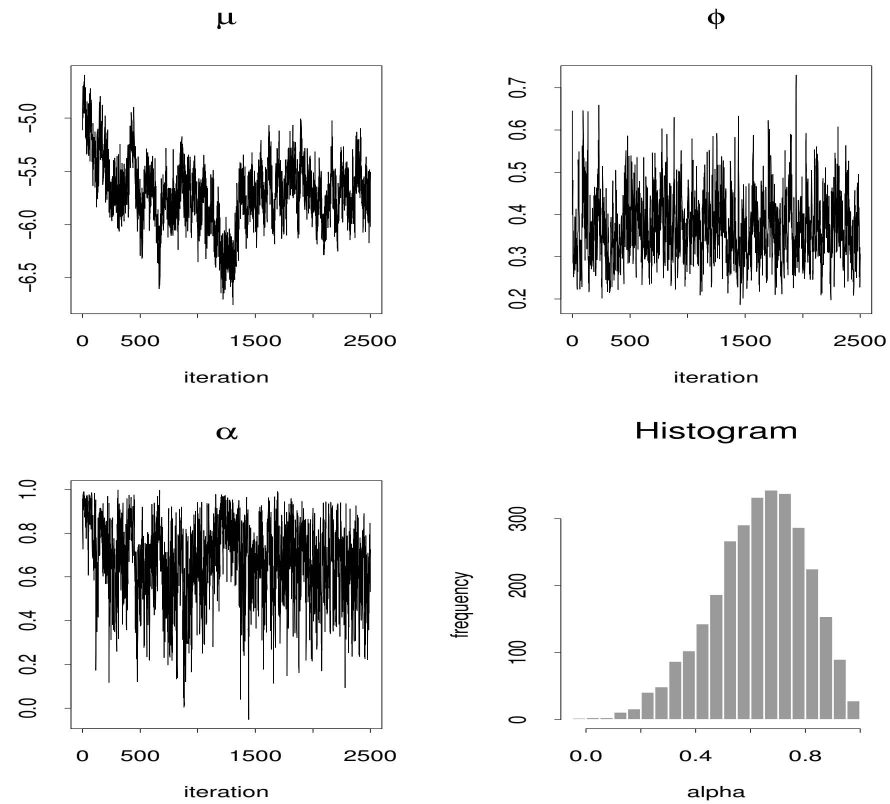
where $g(x) = 1/(1 + \exp\{-x\})$ is the logit response.



State space model for the sequence $(\mathbf{M}_t)$ of default counts.

Results

	μ_{A}	μ_{BBB}	μ_{BB}	μ_{B}	μ_{CCC}	ϕ	α
mean	-9.097	-7.144	-5.712	-3.872	-1.593	0.396	0.649
(std.)	(0.654)	(0.356)	(0.276)	(0.239)	(0.245)	(0.083)	(0.169)



Extended models

Time heterogeneity: (x_t) denotes the Chicago Fed National Activity Index:

Model 2: $M_{tk} \mid \Psi_t = \psi_t \sim B(m_{tk},\, g(\mu_k - \beta x_t - \psi_t))$.

Heterogeneity among rating classes:

Model 3: $M_{tk} \mid \Psi_t = \psi_t \sim B(m_{tk},\, g(\mu_k - \phi_k \psi_t))$.

Sector heterogeneity: Consider S industry sectors, and denote by M_{tsk} (m_{tsk}) the number of defaults (companies) for rating class k, period t, and sector s. Set $\mathbf{\Psi}_t = (\Psi_{t1}, \ldots, \Psi_{tS})'$, where $\Psi_{ts} := \Psi_t + \eta_{ts}$ with (Ψ_t) as before and $\eta_{t1}, \ldots, \eta_{tS}$ iid Gaussian.

Model 4: $M_{tsk} \mid \mathbf{\Psi}_t \sim B(m_{tsk},\, g(\mu_k - \Psi_{ts}))$.

The covariance matrix of $\mathbf{b}_t$ is of compound symmetry type.

Some Empirical Conclusions

- Residual, cyclical, latent component in the systematic risk; even after accounting for observed business cycle covariates.

- Implied asset correlations for companies sharing industry sector is 10.5 %, whereas the across-sector counterpart is only 6.0 %. [1]

- Implied asset correlations do not appear to fall monotonely with increasing probability of default.

[McNeil and Wendin, 2003]

[1] A model without sector-specific latent factors yields an overall implied asset correlation of 6.9 % for the same dataset.

3. Modelling Dependent Migrations

1. Migration Models in GLMM Framework

2. Empirical Analysis of S&P Migration Data

3.1 Migration Models in GLMM Framework

Consider a latent factor (random effect) $\boldsymbol{\Psi}_t$ (vector/scalar). Given $\boldsymbol{\Psi}_t$ we assume that $R_{t1}, \ldots, R_{tm_t}$ are conditionally independent with

$$\boxed{P(R_{ti} \leq \ell \mid \boldsymbol{\Psi}_t = \boldsymbol{\psi}_t) = g\left(\mu_{\kappa(t,i),\ell} - \mathbf{x}'_{ti}\boldsymbol{\beta} - \mathbf{z}'_{ti}\boldsymbol{\psi}_t\right)}$$

where

- $g : \mathbb{R} \rightarrow (0, 1)$ is an increasing response function, e.g. $\Phi(x)$ (ordered probit) or $1/(1 + e^{-x})$ (ordered logit),
- $\kappa(t, i)$ is the rating of obligor i at the outset of period t,
- the intercepts $(\mu_{k,\ell})$ and regressor coefficients β are unknown parameters satisfying $-\infty = \mu_{k,-1} \leq \mu_{k,0} \leq \cdots \leq \mu_{k,K} = \infty$,

GLMMs for Ordered Categorical Responses

$$\boxed{P(R_{ti} \leq \ell \mid \boldsymbol{\Psi}_t = \boldsymbol{\psi}_t) = g\left(\mu_{\kappa(t,i),\ell} - \mathbf{x}_{ti}'\boldsymbol{\beta} - \mathbf{z}_{ti}'\boldsymbol{\psi}_t\right)}$$

- $\mathbf{x}_{ti}$ and $\mathbf{z}_{ti}$ are additional covariates other than rating,
- $\boldsymbol{\Psi}_t$ are latent factors with hyperparameters θ.

This defines a generalized linear mixed model (GLMM) for the ordered, categorical responses $R_{t1}, \ldots, R_{tm_t}$. We refer to $\mathbf{x}_{ti}'\boldsymbol{\beta} + \mathbf{z}_{ti}'\boldsymbol{\Psi}_t$ as the systematic risk of obligor i.

Conditional transition probabilities:

$$P(R_{ti} = \ell \mid \boldsymbol{\psi}_t) = P(R_{ti} \leq \ell \mid \boldsymbol{\psi}_t) - P(R_{ti} \leq \ell - 1 \mid \boldsymbol{\psi}_t).$$

Rating Transitions as Multinomial Trials

Define the rating indicator $\mathbf{Y}_{ti} := \left(I_{\{R_{ti}=0\}}, \ldots, I_{\{R_{ti}=K\}}\right)'$.

Conditional on $\mathbf{\Psi}_t$, we have $\mathbf{Y}_{t1}, \ldots, \mathbf{Y}_{tm_t}$ conditionally independent with

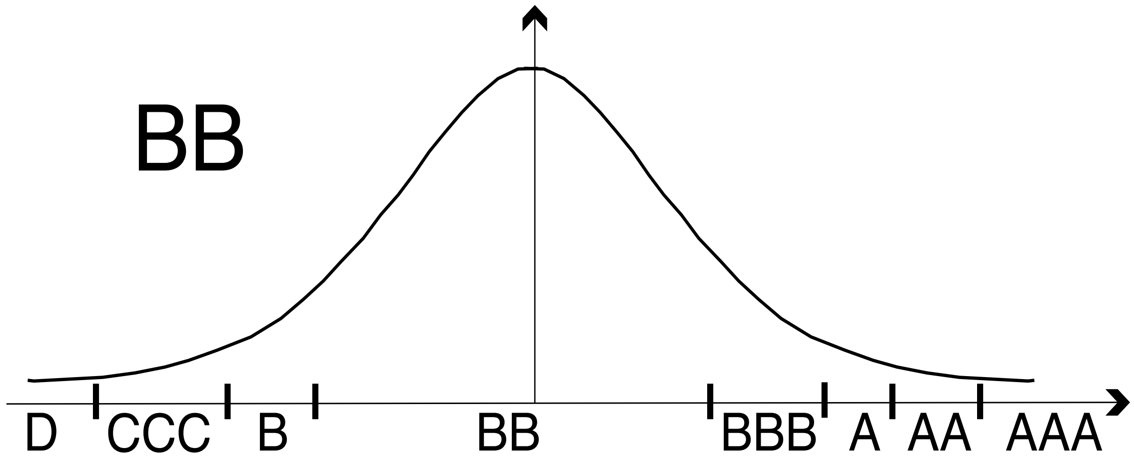
$$\mathbf{Y}_{ti} \mid \mathbf{\Psi}_t = \boldsymbol{\psi}_t \sim \text{Multinomial}\left(1, \mathbf{p}_{\kappa(t,i)}(\mathbf{x}_{ti}'\beta + \mathbf{z}_{ti}'\boldsymbol{\psi}_t)\right),$$

where

$$\mathbf{p}_k(z) = (p_{k,0}(z), \ldots, p_{k,K}(z))' \quad \text{with}$$
$$p_{k,\ell}(z) = g(\mu_{k,\ell} - z) - g(\mu_{k,\ell-1} - z).$$

The elements of the matrix $\operatorname{corr}(\mathbf{Y}_{ti}, \mathbf{Y}_{tj})$ are migration correlations of obligors i and j.

Interpretation as an Asset Value Model



Let $\varepsilon_{t1}, \ldots, \varepsilon_{tm_t}$ be iid rvs with df g (independent of $\mathbf{\Psi}_t$). We define $V_{ti} := \mathbf{x}'_{ti}\beta + \mathbf{z}'_{ti}\mathbf{\Psi}_t + \varepsilon_{ti}$, $i = 1, \ldots, m_t$, and notice that

$$R_{ti} = \ell \quad \Longleftrightarrow \quad V_{ti} \in \big(\mu_{\kappa(t,i),\ell-1},\, \mu_{\kappa(t,i),\ell} \big] \,.$$

Interpretation: V_{ti} is the asset value and $(\mu_{\kappa(t,i),\ell})$ are critical liability levels. We refer to $\mathrm{corr}(V_{ti}, V_{tj})$ as the implied asset correlation between obligors i and j.

Homogeneity within Rating Classes

Denote by $\mathcal{K} = \{1, \ldots, K\}$ the set of non-default credit ratings. Assume that $\mathbf{x}_{ti} := \widetilde{\mathbf{x}}_{tk}$ and $\mathbf{z}_{ti} := \widetilde{\mathbf{z}}_{tk}$ for all obligors with rating $\kappa(t,i) = k$. The obligors in rating class k are thus modelled exchangeably. Introduce

$$\mathbf{M}_{tk} = (M_{t:k,0}, \ldots, M_{t:k,K})' := \sum_{i=1}^{m_t} I_{\{\kappa(t,i)=k\}} \mathbf{Y}_{ti}.$$

Let $m_{tk} = \sum_{i=1}^{m_t} I_{\{\kappa(t,i)=k\}}$ be the number of k-rated obligors in period t. Clearly, we have $\mathbf{M}_{t1}, \ldots, \mathbf{M}_{tK}$ conditionally independent, given $\mathbf{\Psi}_t$, with

$$\mathbf{M}_{tk} \mid \mathbf{\Psi}_t = \boldsymbol{\psi}_t \sim \mathrm{Multinomial}\left(m_{tk}, \mathbf{p}_k(\widetilde{\mathbf{x}}'_{tk}\boldsymbol{\beta} + \widetilde{\mathbf{z}}'_{tk}\boldsymbol{\psi}_t)\right).$$

3.2 Empirical Analysis of S&P Migration Data

Data from S&P: 5,651 US and Canadian firms from 12 industry sectors. The rating classes under consideration are

$$\mathcal{K} = \{\text{CCC}, \text{B}, \text{BB}, \text{BBB}, \text{A}, \text{AA}, \text{AAA}\}$$

Transition count vectors are collected on a three-month basis during 20 years (i.e. $T = 80$ time periods).

We fit several GLMM-type models of increasing complexity by Gibbs sampling with non-informative priors. The latent factors are assumed to follow autoregressive time series.

Our study employs the logit response function $g(x) = 1/(1 + e^{-x})$.

Practical Example

Given (Ψ_t), we assume that $\{\mathbf{M}_{tk} : k \in \mathcal{K}, 1 \le t \le T\}$ are conditionally independent with

$$\mathbf{M}_{tk} \mid \Psi_t = \psi_t \sim \text{Multinomial}\left(m_{tk}, \mathbf{p}_k(x_t\beta + \psi_t)\right),$$

where x_t is the Chicago Fed National Activity Index (CFNAI) for the first month of period t.

The latent factors (Ψ_t) are assumed first-order autoregressive (AR(1)):

$$\Psi_t = \alpha\Psi_{t-1} + \phi\epsilon_t, \quad b_0 = \phi\epsilon_0/\sqrt{1-\alpha^2},$$

where $\epsilon_0, \epsilon_1, \ldots$ are iid $N(0,1)$.

Parameters to estimate are: threshold values $(\mu_{k,\ell})$, regression coefficient β and hyperparameters ϕ and α.

Results

k	Threshold parameters $\mu_{k,\ell}$								
	$\mu_{k,\mathrm{D}}$		$\mu_{k,\mathrm{CCC}}$		$\mu_{k,\mathrm{B}}$		$\mu_{k,\mathrm{BB}}$		$\mu_{k,\mathrm{BBB}}$
AAA	-28.94	(4.78)	-22.61	(5.70)	-16.32	(5.04)	-8.61	$(.84)$	$\cdots$
AA	-11.28	(1.38)	-9.79	$(.80)$	-8.08	$(.38)$	-7.66	$(.32)$	$\cdots$
A	-10.08	$(.68)$	-9.75	$(.60)$	-7.62	$(.24)$	-6.48	$(.15)$	$\cdots$
BBB	-8.15	$(.34)$	-7.73	$(.28)$	-6.18	$(.16)$	-4.22	$(.11)$	$\cdots$
BB	-6.60	$(.20)$	-5.88	$(.15)$	-3.67	$(.10)$	4.13	$(.11)$	$\cdots$
B	-4.69	$(.12)$	-3.72	$(.11)$	4.25	$(.11)$	6.35	$(.17)$	$\cdots$
CCC	-2.04	$(.12)$	3.29	$(.15)$	4.93	$(.27)$	5.71	$(.39)$	$\cdots$
	Remaining parameters								
	β		ϕ		α				
	0.060	$(.051)$	0.256	$(.030)$	0.672	$(.113)$			

Posterior mean (standard deviation). Gibbs sampling with 2,000 iterations (burn-in 5,000). Implied asset correlation: 3.5 %.

Three-month Transition Probability Matrix (%)

$k \backslash \ell$	AAA	AA	A	BBB	BB	B	CCC	D
AAA	97.73	2.04	0.17	0.03	0.02	0.00	0.00	0.00
AA	0.17	97.89	1.82	0.07	0.02	0.03	0.00	0.00
A	0.02	0.49	97.91	1.42	0.11	0.05	0.00	0.00
BBB	0.00	0.06	1.27	97.13	1.31	0.17	0.02	0.03
BB	0.01	0.02	0.12	1.54	95.70	2.33	0.15	0.14
B	0.00	0.02	0.08	0.09	1.30	96.01	1.54	0.96
CCC	0.01	0.01	0.09	0.24	0.41	3.03	84.24	11.97
D	0	0	0	0	0	0	0	100.00

General Empirical Conclusions of Paper

- Residual, cyclical, latent component in the systematic risk; even after accounting for observed business cycle covariates.
- Evidence of heterogeneity between industry sectors. (Multi-factor models can address such concentration risk.)
- Implied asset correlations do not appear to fall monotonically with decreasing credit quality.
- Implied asset correlations intrinsic in transition data appear to be lower than those intrinsic in pure default data.

[Wendin and McNeil, 2004]

References

[Clayton, 1996] Clayton, D. (1996). Generalized linear mixed models. In Gilks, W., Richardson, S., and Spiegelhalter, D., editors, *Markov Chain Monte Carlo in Practice*, pages 275–301. Chapman & Hall, London.

[McNeil and Wendin, 2003] McNeil, A. and Wendin, J. (2003). Generalised linear mixed models in portfolio credit risk modelling. Preprint, ETH Zurich.

[Merton, 1974] Merton, R. (1974). On the pricing of corporate debt: The risk structure of interest rates. *J. Finance*, 29:449–470.

[Robert and Casella, 1999] Robert, C. and Casella, G. (1999). *Monte Carlo Statistical Methods*. Springer, New York.

[Wendin and McNeil, 2004] Wendin, J. and McNeil, A. (2004). Dependent credit migrations. Preprint, ETH Zurich.