SOME NEW RESULTS ON THE INTEGRAL OF GEOMETRIC BROWNIAN MOTION AND THE PRICING OF ASIAN OPTIONS

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ABSTRACT: The subject of the talk is the distribution law of $\int_0^\infty V(W_s + \mu\, s)ds$ and the joint law of $\left(\int_0^t V(W_s)ds, W_t\right)$, with $(W_t)_{t\geqslant 0}$ taken to be some standard Brownian motion, for certain choices for the parameter $\mu \in \mathbb{R}$ and for the function $V\colon \mathbb{R} \mapsto \mathbb{R}$. These laws play an important role in many areas of analysis, physics, probability theory, statistics and, in the special case $V(x) := e^{\sigma\, x}$, $x \in \mathbb{R}$, mathematical finance and actuarial science. Somewhat unexpectedly, integrals of exponential Brownian motion feature also in the analysis of hyperbolic spaces – see [3]. The most intriguing aspect of the "theory" of such functionals is that it connects some seemingly unrelated domains – for example, it allows one to develop some useful exponential counterparts of Levy's and Pitman's theorems (see [4]). Furthermore, this "theory" leads to some striking identities about exponential functionals of Brownian bridge and Brownian motion processes (see [1]).

While the distribution laws of integral functionals of Brownian motion have been studied for quite some time (see [5] and [2]), tractable expressions for the associated densities are still difficult to obtain (usually the densities are characterized in terms of the inverse Laplace transform in the time domain, which, by way of Lamperti's representation, leads to the renowned integral formula obtained by M. Yor [7]). The main objective of the talk is to present a new approach to the study of the distribution law of the integral of geometric Brownian motion. In particular, we will show that M. Yor's formula [7] is the least tractable member of a cluster of integral formulas. In addition, we will give an independent (and considerably simpler) proof of some striking identities about exponential functionals of Brownian motion and Brownian bridge and will obtain even more identities of the same "striking" type. Finally, we will present an explicit formula for the price of a generic Asian option.

REFERENCES

[1] C. Donati-Martin, H. Matsumoto and M. Yor (2000). *On Striking Identities About the Exponential Functionals of the Brownian Bridge and Brownian Motion*. Endre Cski 65, Period. Math. Hungar., **41** no. 1-2, pp: 103-119.
[2] Daniel Dufresne (1990). *The Distribution of a Perpetuity with Application to Risk Theory and Pension Funding*. Scand. Act. Journal, **90**, pp: 39-79.
[3] N. Ikeda and H. Matsumoto (1999). *Brownian Motion on the Hyperbolic Plane and Selberg Trace Formula*. J. Funct. Anal., **163**, pp: 63-110.
[4] H. Matsumoto and M. Yor(2001). *An Analog of Pitman's $2M - X$ Theorem, II: The Role of the Generalized Inverse Gaussian Laws*. Nagoya Math J., **162**, pp: 65-86.
[5] Mark Kac (1949). *On the Distributions of Certain Wiener Functionals*. Trans. Amer. Math. Soc., **65**, pp: 63-110.
[6] Marc Yor (1980). *Loi de L'indice du Lacet Brownien, et Distribution de Hartman-Watson*. Zeit. fr Wahr. und Verw. Gebiete, **53** no. 1, pp: 71-95.
[7] Marc Yor (1992). *On Some Exponential Functionals of Brownian Motion*. Adv. Appl. Prob., **24** pp: 509-531.

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