# Estimating Default Probabilities Implicit in Equity Price

**Tibor Janosi**
Computer Science
Department
Cornell University

**Robert Jarrow**
Johnson Graduate School
of Management
Cornell University

**Yildiray Yildirim**
School of
Management
Syracuse University

QMF2001

## Goal

To estimate default probabilities using equity prices in conjunction with a reduced form modeling approach.

# Results

First, the best performing intensity model depends on the spot rate of interest but not an equity market index.

Second, due to the large variability of equity prices, the point estimates of the default intensities obtained are not very reliabl

Third, we find that equity prices contain a bubble component n captured by the Fama-French (1993,1996) four-factor model fc equity's risk premium.

Fourth, we compare the estimates of the intensity process obtained here with those obtained using debt prices from Janosi, Jarrow, Yildirim (2000) for the same fifteen firms over the same time period. The hypothesis that these two intensity functions are equivalent cannot be rejected.

# Literature Review

**odels**

<u>Structural Model:</u> Merton' 74

<u>Reduced Form Models:</u> Duffie, Singleton' 94
Jarrow, Turnbull' 95

<u>Combination:</u> Duffie, Lando' 01
Cetin, Jarrow, Protter, Yildirim '01

<u>Review Paper:</u> Bilecki and Rutkowski' 00

**stimation:**

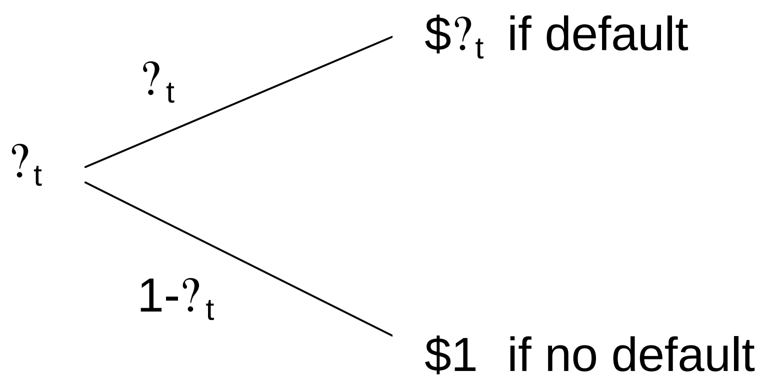
' Altman' 68 and Zmijewski' 84 ? Static Models

' Shumway' 01 and Jarrow, Yildirim, Wells' 01 ? Hazard Models

' Duffee' 99 and Janosi, Jarrow, Yildirim' 01 ? Estimation based on reduced form model

' Delianedis and Geske' 98 ? Estimation based on equity prices

## mple Binomial Reduced Form Model




$$v_t ? e^{?r_t}?\,?_t d_t ? 1 ? ?_t ? ? e^{?r_t} e^{? ?_t (1?d_t)} ? e^{?(r_t ? ?_t(1?d_t))}$$

(Note that $e^{c} ? 1 ? c$ for small c)

## Reduced Form Credit Risk Model For Equity Prices

Frictionless markets with no arbitrage opportunities, but equity prices may contain bubbles.

aded are:

- Default-free zero-coupon bonds of all maturities and
- Firm's common stock

Let

$$N(t) ? 1_{?t ? t?} ? \begin{cases} ?1 & if \quad t ? t \\ ? \\ ?0 & otherwise \end{cases}$$

denote the point process indicating whether or not default has occurr
prior to time *t*.

$?(t)$ denotes its random intensity process.

Equity pays

1. "coupons", called dividends $D_t$ , paid at *1, 2, ... ,* $T_D^*$
2. $L(T_L^*)$ : liquidating payoff at time $T_L^*$ for $0 ? T_L^* ? T$

*S(t)*: the time t present value of the liquidating dividend, conditiona
upon no default prior to time t.

$?(t)$ : "bubble" component.

$?(t)$ : time t value of these promised payments per share – the value of the equity

If default occurs, the equity holders get a fractional recovery payment on these promises equal to $?_e(?)?(??)$ where $?_e(?)$ ?

Given this set-up, it is easy to see that the per share equity value at time t is given by

$$?(t) ? \begin{cases} S(t) ? ?(t) ? \sum_{j ? t}^{T_D^*} D_j \, p(t,j) & \text{if } t ? ? \\ 0 & \text{if } t ? ? . \end{cases}$$

## sk Neutral Valuation

nder the measure Q,

$$p(t,T) ? E_t(e^{? ?_t^T r(u)du})$$

$$S(t) ? E_t(L(T_L^*)e^{? ?_t^{T_L^*} r(u)du} 1_{(T_L^* ??)})$$

or simplicity, we model bubble as follows:

$$? (t) ? S(t)(e^{?_0^t ? ? (u)du} ? 1)$$

To obtain an empirical formulation of above model, more structure needs to be imposed on the stochastic nature of the economy.

Consider an economy that is Markov in three state variables:

- Spot rate of interest
- The cumulative excess return on an equity index
- Liquidating dividend process

*(Spot Rate Evolution)*

- Single factor HJM model with deterministic volatilities, sometimes called the extended Vasicek Model.
- The structure evolution under risk neutral measure is:

$$r(t) = f(0,t) + \frac{\sigma_r^2(e^{-at} - 1)}{2a^2} + \int_0^t \sigma_r e^{-a(t-u)} dW(u)$$

*(Market Index Evolution)*

- Geometric Brownian motion with drift r(t) and volatility $\sigma_m$

$$dM(t) = M(t)\left[r(t)dt + \sigma_m dZ(t)\right]$$

$dZ(t)dW(t) = \rho_{rm}(t)$ with $\rho_{rm}$ a constant

*(Liquidation Value Evolution)*

- Geometric Brownian motion with drift r(t) and volatility $?_L???$

$$dL(t) \, ? \, r(t)L(t)dt \, ? \, ? \,_L L(t)dw_L(t)$$

$dZ(t)dw_L(t) = ?_{mL}$ *and* $dw_L(t)dW(t) = ?_{rL}$ *with* $?_{mL}$ and $?_{rL}$ constants

Given the evolutions of the state variables, we next specify the intensity process.

*(Intensity a Function of the Spot Rate and the Market Index )*

$$?(t) ? \max[?_0 ? ?_1 r(t) ? ?_2 Z(t),\ 0]$$

*where* $?_0, ?_1, ?_2$ are constants.

Given these expressions, the default free zero-coupon bond ar
the present value of the liquidating dividend can be rewritten as

$$p(t,T) ? e^{? ?_1(t,T) ?? ?_1^2(t,T)/2}$$

$$? \frac{L(t)}{p(t,T_L^*)} e^{??_1^? \,_1^2(t,T_L^*)??_1^? L^? rL \int_t^{T_L^*} b(u,T_L^*)du ??_2^? rm^? (t,T_L^*)??_2^? L^? mL (T_L^* ? t)^2/2} \quad v(t,T$$

where

$$? p(t,T) e^{??_0 (T?t)??_1^? \,_1(t,T)?[2?_1??_1^2 ? \,_1^2(t,T)/2??_2 Z(t)(T?t)?[1??_1 ?]_2^? rm^? (t,T)?[T?$$

$$? \int_t^T f(t,u)du \; ? \int_t^T b(u,T)^2 du/2, \; ? \,_1^2(t,T) \; ? \int_t^T b(u,T)^2 du,$$

$$? ? _r [1 ? e^{?a(t?u)}]/a, \quad and$$

$$? ? (? _r /a^3)[1 ? e^{?a(T?t)}] ? (? _r /a^2) e^{?a(T?t)} (T ? t) ? (? _r /2a)[T ? t]^2$$

Unfortunately, this system under determined as there are more unknowns (*L(t*??? ?,? ?,? ? ? than there are observables ??(t)??

To overcome this situation, we use *Liquidation Value Evolution* in conjunction with expression above to transform ?(t) expression into a time series regression.

$$\log\left[\frac{?(t) ? \sum_{j?t}^{T_D^*} D_j p(t,j)}{?(t??) ? \sum_{j?t??}^{T_D^*} D_j p(t??,j)}\right] ? r(t??)?$$

$$? ??_0 ? \quad ??_1 \left[\frac{b(t??,T_L^*)^2}{2}\right] ? ? \log\left[\frac{p(t,T_L^*)}{p(t??,T_L^*)}\right]$$

$$? ?_2 [Z(t)(T^* ? t) ? Z(t??)(T^* ? t??)]$$

$$? ?_1 ?_2 ? \, rm \, b(t,T_L^*)(T_L^* ? t)? \quad ? ? \frac{2}{2}(T^* ? t)^2 ? /6$$

$$? [? _L ? _L (t??) ? ? _L^2 ? ?_? (t??) ? (1/2) ?_?^2]?$$

$$? ?(t??)$$

where

$?(t??) ? ?_L(w_L(t) ? w_L(t??))$ and $?_L(t)$ is the liquidation value's risk premium

# Data Description

Firm equity data are obtained from CRSP.

For equity market index, the S&P 500 index is used.

For estimating an equity risk premium, we will employ the Fam French benchmark portfolios (book-to-market factor (HML), small firm factor (SMB)), and a momentum factor (UMD). Thes monthly portfolio returns were obtained from Ken French's webpage

U.S. Treasury securities are obtained from University Houston' Fixed Income Database.

The time period covered: May 1991-March 1997.

The same twenty firms as in Janosi, Jarrow, Yildirim (2000) were initially selected for analysis.

| | Ticker Symbol | SIC Code | First Date used in the Estimation | Last Date used in the Estimation | Number of Bonds | Moodies | S&P |
|---|---|---|---|---|---|---|---|
| **Financials** | | | | | | | |
| BANKERS TRUST NY | bt | 6022 | 01/31/1994 | 04/30/1994 | 3 | A1 | AA |
| MERRILL LYNCH & CO | mer | 6211 | 12/31/1991 | 03/31/1997 | 14 | A2 | A |
| **Food & Beverages** | | | | | | | |
| PEPSICO INC | pep | 2086 | 12/31/1991 | 03/31/1997 | 8 | A1 | A |
| COCA - COLA ENT. INC | cce | 2086 | 12/31/1991 | 06/30/1994 | 3 | A2 | AA- |
| **Airlines** | | | | | | | |
| AMR CORPORATION | amr | 4512 | 02/29/1992 | 08/31/1994 | 2 | Baa1 | BBB + |
| SOUTHWEST AIRLINES | luv | 4512 | 05/31/1992 | 03/31/1997 | 3 | Baa1 | A- |
| **Utilities** | | | | | | | |
| CAROLINA POWER LIGHT | cpl | 4911 | 08/31/1992 | 01/31/1993 | 3 | A2 | A |
| TEXAS UTILITIES ELE CO | txu | 4911 | 04/30/1994 | 03/31/1997 | 4 | Baa2 | BBB |
| **Petroleum** | | | | | | | |
| MOBIL CORP | mob | 2911 | 12/31/1991 | 02/29/1996 | 3 | Aa2 | AA |
| **Department Stores** | | | | | | | |
| SEARS ROEBUCK + CO | s | 5311 | 12/31/1991 | 08/31/1996 | 7 | A2 | A |
| WAL-MART STORES, INC | wmt | 5331 | 12/31/1991 | 03/31/1997 | 3 | Aa3 | AA |
| **Technology** | | | | | | | |
| EASTMAN KODAK COMPANY | ek | 3861 | 01/31/1992 | 09/30/1994 | 3 | A2 | A- |
| XEROX CORP | xrx | 3861 | 12/31/1991 | 03/31/1997 | 4 | A2 | A |
| TEXAS INSTRUMENTS | txn | 3674 | 10/31/1992 | 03/31/1997 | 3 | A3 | A |
| INTL BUS MACHINES | ibm | 3570 | 01/31/1994 | 03/31/1997 | 3 | A1 | AA- |

**Table 1: Details of the Firms Included in the Empirical Investigation.**

# Estimation of the state variable process parameters

*(Spot Rate Process Parameter Estimation)*

The inputs to the spot rate process evolution:

- the forward rate curves (f(t,T) for all months t ? January 1975 – March 1997)
- the spot rate parameters (a, $?_r$).

stimation of the forward rate curve) ✍ two-step procedure is utili

1) for a given time *t*,

choose (*p(t,T)* for all relevant $T ? \max\{T_i : i ? I_t\}$)

to minimize $\underset{i ? I_t}{?} ? B_i(t,T_i) ? B_i(t,T_i)^{bid} ?^2$

2) fit a continuous forward rate curve to the estimated zero-coupon bor prices (*p(t,T)* for all $T ? \max\{T_i : i ? I\}$) - Janosi' 00

?? For ? = *1/12* (a month), the expression is:

$$\mathrm{var}_t[\log(P(t ? ?,T)/P(t,T)) ? r(t)? ] ? ?? \; ?^2_{rt} ? e^{?a_t(T?t)} ? 1 ?^2 / a_t^2 ?? \; .$$

?? compute the sample variance, denoted $v_{tT}$, using T ? {3 months, 6 mo year, 5 years, 10 years, 30 years}

?? estimate the parameters ($?_{rt}$,$a_t$)

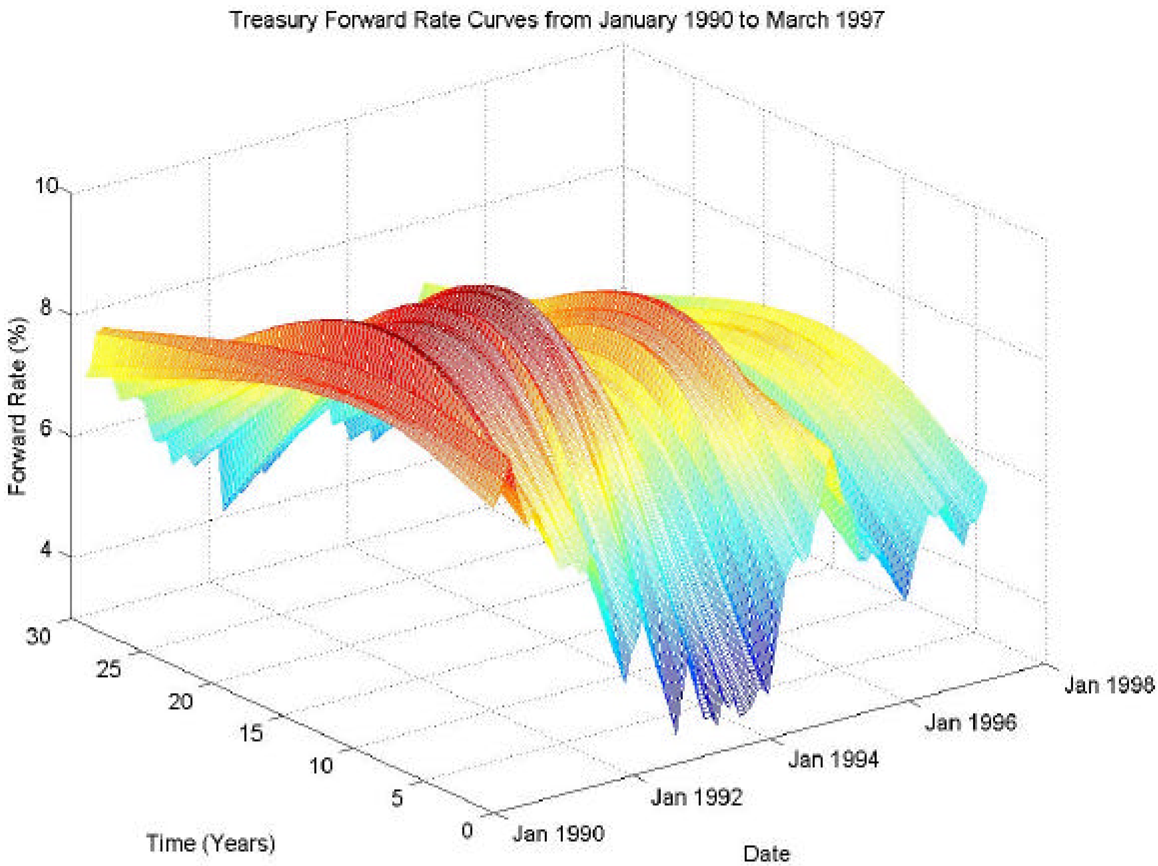



**Figure 1: Treasury Forward Rate Curves from January 1990 to March 1997.**

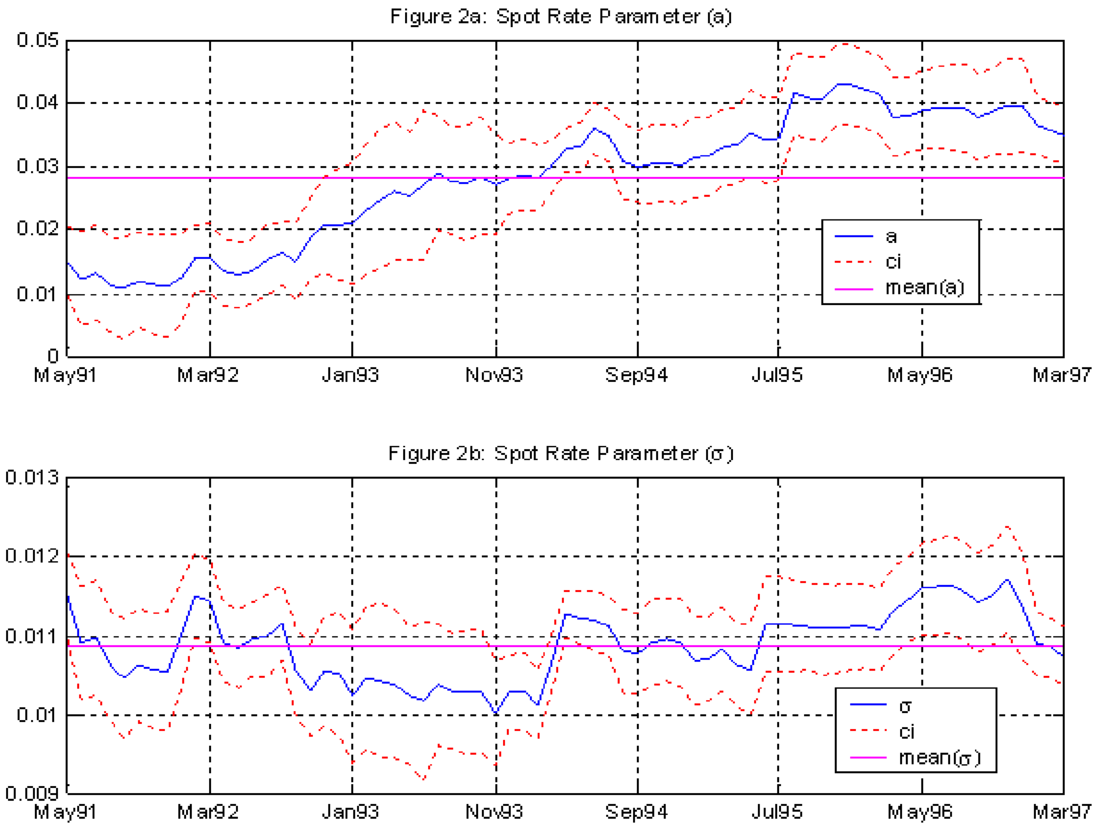



**Figure 2: Time Series Estimates of the Spot Rate Parameters (*a*, ? $_r$) with 90 percent Confi Bands from May 1991 – March 1997.**

*(Market Index Parameter Estimation)*

Using the daily S&P 500 index price data and the 3-month T-bi spot rate data:

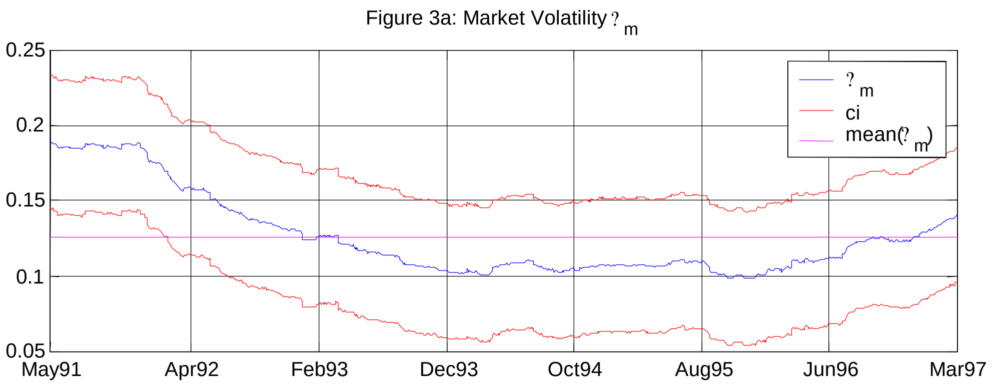
- the parameters of the market index process ✍ (? $_m$,? $_r$
- the cumulative excess return on the market index ✍ Z(t)

For a given date t ? {May 24,1990 – March 31,1997}

- go back in time 365 business days and estimate the time dependent sample variance and correlation coefficients (? $_{mt}$,? $_{rmt}$) using the sample moments:

$$? \, {}^{2}_{mt} \; ? \; \mathrm{var}_t \, ? \frac{M(t) \, ? \, M(t \, ? \, ?)}{M(t \, ? \, ?)} ? \frac{1}{?} \quad \textit{and}$$

$$? \, {}_{rmt} \; ? \; corr_t \, ? \frac{M(t) \, ? \, M(t \, ? \, ?)}{M(t \, ? \, ?)}, r(t) \, ? \, r(t \, ? \, ?) ?$$



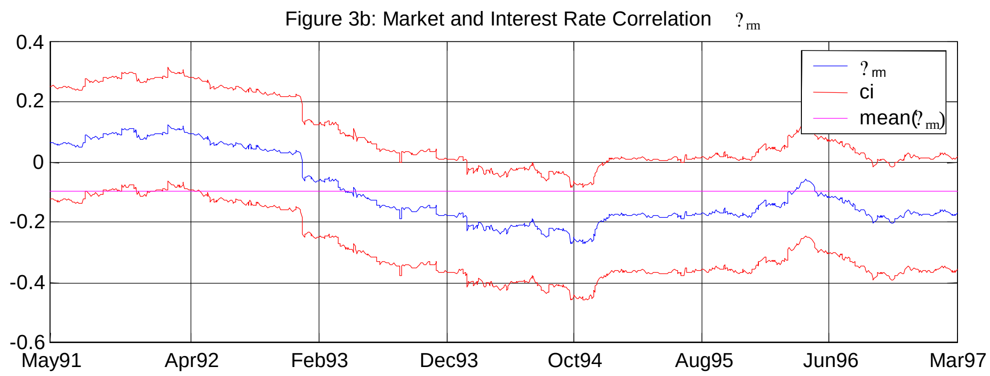




**Figure 3: Time Series Estimates of the Market Index Parameters (? m,? rm) with 90 percent Confidence Bands from May 1991 – March 1997.**

# Equity Model Parameter Estimation

To empirically implement the previous expression, we need to specify models for both the risk premium and the equity price bubble.

Following Fama and French (1993, 1996) we use a four-factor asset pricing model with the factors being the excess return on market portfolio, *SMB(t)*, *HML(t)* and *UMD(t)* , i.e.

$$? _L ? _L (t ? ? , X(t ? ?)) ? ?$$

$$? _0 \left( \frac{M(t) ? M(t ? ?)}{M(t ? ?)} ? r(t ? ?) \right) ? ? _1 SMB(t) ? ? _2 HML(t) ? ? _3 UMD$$

The equity price bubble and the volatility of *L(t)*, ? (t) are proxie by the volatility of the stock and a price earnings ratio,

$$[??\,{}^{2}_{L}\,??_{?}\,(t\,??,X(t\,??))]\,??\,(1/2)?^{2}_{?}(t)\,??\,{}_{4}\,?^{2}_{?}(t)\,??\,{}_{5}\,\frac{Price}{Earnings}(t)$$

where $?^{2}_{?}(t)\,?\,\text{var}\left(\frac{?(t)\,??\,(t\,??)}{?\,(t\,??)}\right)\frac{1}{?}$

There is some concern that the *SMB(t)*, *HML(t)* and *UMD(t)* factors may already include an adjustment for bubbles. For thi reason, the subsequent regressions are run both with and without the P/E ratio included.

Substitution of the above into our original expression yields:

$$\log\left(\frac{?(t)}{?(t??)}\right) ? r(t??)? \quad \textit{if no dividend over } [t??,t]$$

$$\log\left(\frac{?(t)}{?(t??)?D_x p(t??,x)}\right) ? r(t??)? \quad \textit{if dividend at } x?[t??,t]$$

$$??_0 ??_1\left(\log\left(\frac{p(t,T_L^*)}{p(t??,T_L^*)}\right)?\left(\frac{b(t??,T_L^*)^2}{2}\right)\right)??_2\left(Z(t)(T_L^*?t)?Z(t??)(T_L^*?(t\right.$$

$$??_0\left(\frac{M(t)?M(t??)}{M(t??)}?r(t??)?\right)??_1 SMB(t)??_2 HML(t)??_3 UMD(t)$$

$$??_4\left(?^2(t)?\right)??_5\left(\frac{\text{Price}}{\text{Earnings}}(t)\right)$$

where

$?_0 ? ??_0? ??_1?_2? rm b(t,T_L^*)(T_L^*?t)? ??\frac{2}{2}(T_L^*?t)^2? /6$

$?_1 ? ?_1$

$?_2 ? ??_2$.

The time period covered is May 1991 – March 1997.

The estimating regression is run using 48 months of historical data.

Thus, the first regression estimation occurs four years into the data set on May 31, 1995.

For each subsequent month, until March 1997, the regression re-estimated and parameter estimates obtained. This generate 23 regressions. As before, only information available to the market at the time of the estimation is utilized.

This rolling estimation procedure gives a time series of parameter estimates $(?_{0t}, ?_{1t}, ?_{2t}, ?_{0t}, ?_{1t}, ?_{2t}, ?_{3t}, ?_{4t}, ?_{5t})$ based on 46 (48 – 2) months of overlapping data.

INTERNATIONAL BUSINESS MACHINES

| ͻM | ? $_0$ | ? $_1$ | ? $_2$ | ? $_0$ | ? $_1$ | ? $_2$ | ? $_3$ | ? $_4$ | ? $_5$ |
|---|---|---|---|---|---|---|---|---|---|
| del 1 | | | | 0.9832 | -0.0092 | -0.0076 | -0.0113 | 0.4104 | |
| del 2 | | | | 1.0009 | -0.0092 | -0.0075 | -0.0103 | 0.1364 | -0.0016 |
| del 3 | 0.0000 | | | 0.9832 | -0.0092 | -0.0076 | -0.0113 | 0.4104 | |
| del 4 | 0.0000 | | | 1.0011 | -0.0092 | -0.0075 | -0.0103 | 0.1375 | -0.0016 |
| del 5 | 0.0000 | -0.6426 | | 1.5795 | -0.0110 | -0.0030 | -0.0079 | 0.2092 | |
| del 6 | 0.0000 | -0.6361 | | 1.5908 | -0.0111 | -0.0030 | -0.0070 | -0.0300 | -0.0015 |
| del 7 | 0.0000 | -0.6643 | 0.0081 | 3.1913 | -0.0113 | -0.0028 | -0.0074 | 0.1624 | |
| del 8 | 0.0000 | -0.6571 | 0.0084 | 3.2470 | -0.0113 | -0.0028 | -0.0065 | -0.0921 | -0.0015 |

## ble 3: Averages of the Parameter Estimates from the Equity Model gression.

ıle 3 contains the average parameter estimates *(? $_0$ ,? $_1$ ,? $_2$ ,? $_0$ ,? $_1$ ,? $_2$ ,? $_3$ ,? $_4$ ,? $_5$ )* acro ıths in the observation period from the equity model regressions. They are presented h company and for each model type, separated by industries. Models 1 and 2 have nc ault. Models 3 and 4 have a constant default intensity. Models 5 and 6 have the defau nsity dependent on the spot rate of interest. Models 7 and 8 have the default intensity ıendent on the spot rate of interest and a market index. The number of observations p ression is 48. The number of regressions in the average is 23. The average $R^2$ is give

**15-INTERNATIONAL BUSINESS MACHINES**

| ibm | $?_0$ | $?_1$ | $?_2$ | $?_0$ | $?_1$ | $?_2$ | $?_3$ | $?_4$ | $?_5$ | F |
|---|---|---|---|---|---|---|---|---|---|---|
| Model 1 | | | | 2.0200** | -1.465 | -1.2198 | -2.072** | 0.5476 | | ( |
| | | | | 0.0220 | 0.0876 | 0.1405 | 0.0340 | 0.2403 | | |
| Model 2 | | | | 2.0352** | -1.459 | -1.1911 | -1.764** | 0.2812 | -0.5997 | ( |
| | | | | 0.0208 | 0.0894 | 0.1488 | 0.0505 | 0.2215 | 0.2233 | |
| Model 3 | 0.0000 | | | 1.9613** | -1.444 | -1.2010 | -2.012** | 0.3351 | | ( |
| | 0.4038 | | | 0.0253 | 0.0893 | 0.1425 | 0.0383 | 0.2764 | | |
| Model 4 | -0.0011 | | | 1.9720** | -1.438 | -1.1723 | -1.705** | 0.1600 | -0.5885 | ( |
| | 0.4035 | | | 0.0243 | 0.0911 | 0.1507 | 0.0565 | 0.2597 | 0.2262 | |
| Model 5 | 0.0000 | -1.921** | | 2.7590** | -1.793** | -0.4595 | -1.3827 | 0.1929 | | ( |
| | 0.4038 | 0.0378 | | 0.0033 | 0.0569 | 0.2748 | 0.1054 | 0.2897 | | |
| Model 6 | 0.0000 | -1.884** | | 2.7464** | -1.780** | -0.4477 | -1.1595 | 0.0377 | -0.5491 | ( |
| | 0.4038 | 0.0414 | | 0.0034 | 0.0592 | 0.2703 | 0.1349 | 0.2732 | 0.2375 | |
| Model 7 | 0.0000 | -1.970** | -0.0235 | 0.5644 | -1.793** | -0.4291 | -1.2641 | 0.1591 | | ( |
| | 0.4038 | 0.0285 | 0.2053 | 0.2128 | 0.0459 | 0.2898 | 0.1055 | 0.2859 | | |
| Model 8 | -0.0041 | -1.931** | -0.0288 | 0.5652 | -1.777** | -0.4140 | -1.0401 | -0.0028 | -0.5818 | ( |
| | 0.4025 | 0.0314 | 0.2014 | 0.2081 | 0.0478 | 0.2865 | 0.1393 | 0.2682 | 0.2285 | |

**4: T-Scores and Average P-values for the Estimated Parameters from th**
**y Model Regression**

icant at 10% level
cant at 15% level

| | ? 0 | ? 1 | ? 0 | ? 1 | ? 2 | ? 3 |
|---|---|---|---|---|---|---|
| ? 0 | 1.0000 | 0.4685 | 0.4580 | -0.4072 | 0.2607 | 0.2896 |
| ? 1 | 0.4685 | 1.0000 | 0.9934 | -0.1277 | -0.3539 | 0.1141 |
| ? 0 | 0.4580 | 0.9934 | 1.0000 | -0.1449 | -0.3765 | 0.1060 |
| ? 1 | -0.4072 | -0.1277 | -0.1449 | 1.0000 | -0.5153 | 0.0850 |
| ? 2 | 0.2607 | -0.3539 | -0.3765 | -0.5153 | 1.0000 | -0.0267 |
| ? 3 | 0.2896 | 0.1141 | 0.1060 | 0.0850 | -0.0267 | 1.0000 |

**le 5: Correlation Matrix for Selected Independent Variables he Equity Model Regression**

**15-INTERNATIONAL  BUSINESS MACHINES**

| **ibm** | ?$_{0}$ | ?$_{1}$ | ?$_{2}$ | ?$_{0}$ | ?$_{1}$ | ?$_{2}$ | ?$_{3}$ | ?$_{4}$ | ?$_{5}$ |
|---|---|---|---|---|---|---|---|---|---|
| Model 1 | | | | -0.4822 | -1.1758 | 0.0008 | -0.6878 | -0.3619 | |
| | | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | |
| Model 2 | | | | -0.6719 | -1.2155 | -0.1199 | -0.2635 | -0.6033 | -4.6382 |
| | | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0100 |
| Model 3 | 0.0000 | | | -0.4822 | -1.1758 | 0.0008 | -0.6878 | -0.3619 | |
| | 0.0000 | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | |
| Model 4 | 0.0000 | | | -0.6804 | -1.2161 | -0.1201 | -0.2666 | -0.6204 | -4.6311 |
| | 0.0000 | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0100 |
| Model 5 | 0.0000 | -1.7948 | | -1.1128 | -1.1613 | 0.3665 | -1.1714 | -0.3399 | |
| | 0.0000 | 1.0000 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | |
| Model 6 | 0.0000 | -1.8823 | | -1.2496 | -1.1790 | 0.2745 | -1.1204 | -0.6372 | -3.7850 |
| | 0.0000 | 1.0000 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0100 |
| Model 7 | 0.0000 | -1.6365 | -1.2616 | -1.2433 | -1.3287 | 0.1635 | -1.2780 | -0.3107 | |
| | 0.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | |
| Model 8 | 0.0000 | -1.7297 | -1.2780 | -1.2680 | -1.3544 | 0.0675 | -1.2810 | -0.6505 | -3.6836 |
| | 0.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0500 |

Performance Across all Companies

| | ?$_{0}$ | ?$_{1}$ | ?$_{2}$ | ?$_{0}$ | ?$_{1}$ | ?$_{2}$ | ?$_{3}$ | ?$_{4}$ | ?$_{5}$ |
|---|---|---|---|---|---|---|---|---|---|
| Model 1 | | | | 2/15 | 0/15 | 0/15 | 1/15 | 2/15 | 0/15 |
| Model 2 | | | | 2/15 | 1/15 | 0/15 | 1/15 | 3/15 | 2/15 |
| Model 3 | | | | 2/15 | 0/15 | 0/15 | 1/15 | 3/15 | 0/15 |
| Model 4 | | | | 2/15 | 1/15 | 0/15 | 1/15 | 3/15 | 2/15 |
| Model 5 | | 2/15 | | 4/15 | 1/15 | 1/15 | 3/15 | 4/15 | 0/15 |
| Model 6 | | 1/15 | | 4/15 | 0/15 | 1/15 | 3/15 | 3/15 | 2/15 |
| Model 7 | | 2/15 | 3/15 | 3/15 | 2/15 | 1/15 | 3/15 | 4/15 | 0/15 |
| Model 8 | | 0/15 | 4/15 | 3/15 | 3/15 | 0/15 | 3/15 | 3/15 | 3/15 |

**Table 6: Unit Root Tests**

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| IBM | Avg GCV | Avg RMSE | ? | se(?) | 1ydf | Avg Y values | $R^2$ |
|---|---|---|---|---|---|---|---|
| Model 1 | 0.0066 | 0.0769 | | | | 0.0059 | 0.2812 |
| Model 2 | 0.0069 | 0.0775 | | | | 0.0059 | 0.2867 |
| Model 3 | 0.0070 | 0.0778 | 0.0000 | 0.2304 | 0.0000 | 0.0059 | 0.2812 |
| Model 4 | 0.0072 | 0.0784 | 0.0000 | 0.2341 | 0.0003 | 0.0059 | 0.2867 |
| Model 5 | 0.0067 | 0.0752 | -0.0318 | 0.2238 | -0.0372 | 0.0059 | 0.3467 |
| Model 6 | 0.0069 | 0.0758 | -0.0315 | 0.2276 | -0.0368 | 0.0059 | 0.3510 |
| Model 7 | 0.0068 | 0.0753 | 0.0045 | 0.2423 | -0.0340 | 0.0059 | 0.3601 |
| Model 8 | 0.0071 | 0.0759 | 0.0061 | 0.2460 | -0.0326 | 0.0059 | 0.3651 |

| Average | Avg GCV | Avg RMSE | ? | se(?) | 1ydf | Avg Y values | $R^2$ |
|---|---|---|---|---|---|---|---|
| Model 1 | 0.0044 | 0.0597 | | | | 0.0026 | 0.5018 |
| Model 2 | 0.0045 | 0.0595 | | | | 0.0026 | 0.5174 |
| Model 3 | 0.0046 | 0.0603 | 0.0280 | 0.1776 | 0.0259 | 0.0026 | 0.5051 |
| Model 4 | 0.0047 | 0.0600 | 0.0244 | 0.2711 | 0.0331 | 0.0026 | 0.5197 |
| Model 5 | 0.0046 | 0.0599 | 0.0237 | 0.1781 | 0.0208 | 0.0026 | 0.5261 |
| Model 6 | 0.0047 | 0.0596 | 0.0219 | 0.2752 | 0.0293 | 0.0026 | 0.5409 |
| Model 7 | 0.0047 | 0.0598 | 0.0203 | 0.1949 | -0.0135 | 0.0026 | 0.5397 |
| Model 8 | 0.0048 | 0.0595 | 0.0178 | 0.2882 | -0.0066 | 0.0026 | 0.5548 |

## Table 7: Summary Statistics for Model Performance

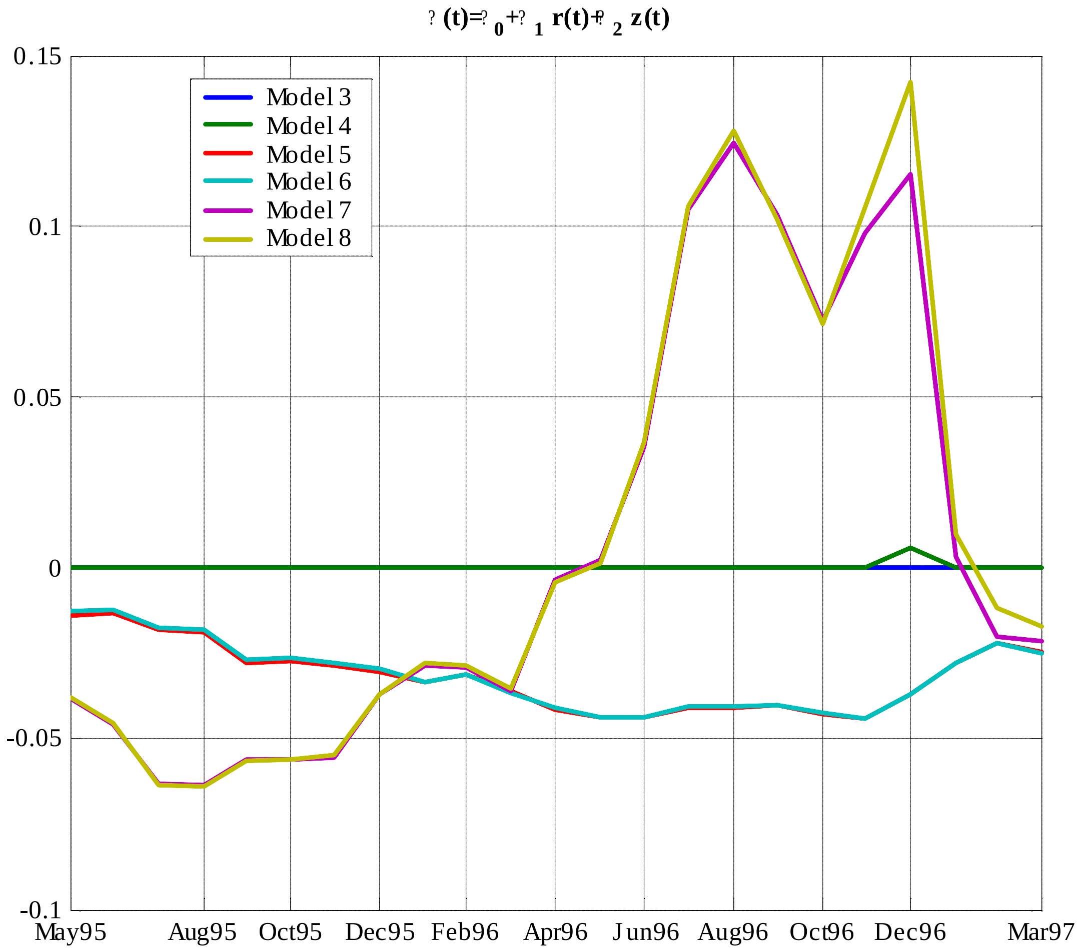


Figure 5: Time Series Estimates of IBM's Intensity Function

# Comparison of default intensities based on debt and equity

Due to non-overlapping periods of observations in the tw studies, only 10 firms are included in this comparison. The fiv firms omitted are: American Airlines, Bankers Trust, Coca-Col Carolina Power Light, Kodak.

Given are:

??Estimated intensities $?_e^i(t)$ from the equity price model and

??Estimated expected losses (per unit time) $a_d^i(t) ? ?_d^i(t)(1 ? ?^i)$

for ten firms. The null hypothesis to be tested is:

(debt) $a_d^i(t)/(1 ? ?^i) ? ?_e^i(t)$ (equity) for all firms $i$ and all times $t$.

| | Avg(? d(t)) | Avg(? e(t)) | Avg(t-score) |
|---|---|---|---|
| **amr** | N/A | N/A | N/A |
| | N/A | N/A | |
| **bt** | N/A | N/A | N/A |
| | N/A | N/A | |
| **cce** | N/A | N/A | N/A |
| | N/A | N/A | |
| **cpl** | N/A | N/A | N/A |
| | N/A | N/A | |
| **ek** | N/A | N/A | N/A |
| | N/A | N/A | |
| **ibm** | 0.0103 | -0.0315 | 0.1826 |
| | 0.0069 | 0.2271 | |
| **luv** | 0.0104 | -0.0092 | 0.0499 |
| | 0.0064 | 0.4995 | |
| **mer** | 0.0100 | 0.0048 | 0.0164 |
| | 0.0029 | 0.3201 | |
| **mob** | 0.0036 | 0.0069 | -0.0148 |
| | 0.0009 | 0.2192 | |
| **pep** | 0.0082 | -0.0001 | 0.0229 |
| | 0.0075 | 0.3649 | |
| **s** | 0.0086 | 0.0023 | 0.0482 |
| | 0.0074 | 0.1233 | |
| **txn** | 0.0083 | -0.0121 | 0.076 |
| | 0.0089 | 0.3003 | |
| **txu** | 0.0061 | 0.0176 | -0.0773 |
| | 0.0067 | 0.1531 | |
| **wmt** | 0.0036 | 0.171 | -0.583 |
| | 0.0045 | 0.3221 | |
| **xrx** | 0.0056 | -0.0101 | 0.1176 |
| | 0.0057 | 0.1507 | |

**Table 8: Test for the Equivalence Between the Default Intensities based on De Prices versus Equity Prices**

Although this evidence is consistent with the equivalence of the two default intensities, the large standard errors for the equity model estimates indicates that this is a weak test of the null hypothesis.

In fact, the evidence is consistent with any recovery rate between 0 and less than one.

We now provide an independent test for the time series stationarity of the recovery rate, estimated as the ratio of the implicit and explicit estimates.

- Under the null hypothesis, the difference between adjacent time series estimates of the ratio is zero for any firm *I*.

$$\frac{\lambda_d^i(t)\left(1-\delta^i(t)\right)}{\lambda_e^i(t)} - \frac{\lambda_d^i(t-1)\left(1-\delta^i(t-1)\right)}{\lambda_e^i(t-1)} = \beta_0 + \beta_1\left[\frac{\lambda_d^i(t-1)\left(1-\delta^i(t-1)\right)}{\lambda_e^i(t-1)} - \frac{\lambda_d^i(t-2)\left(1-\delta^i(t-2)\right)}{\lambda_e^i(t-2)}\right] + \varepsilon$$

| | ? 0 | ? 1 | $R^2$ | N |
|---|---|---|---|---|
| **amr** | NaN | NaN | NaN | NaN |
| | NaN | NaN | | |
| **bt** | NaN | NaN | NaN | NaN |
| | NaN | NaN | | |
| **cce** | NaN | NaN | NaN | NaN |
| | NaN | NaN | | |
| **cpl** | NaN | NaN | NaN | NaN |
| | NaN | NaN | | |
| **ek** | NaN | NaN | NaN | NaN |
| | NaN | NaN | | |
| **ibm** | -0.0028 | -0.5263 | 0.2459 | 21 |
| | -0.1383 | -2.4888* | | |
| **luv** | 0.0557 | -0.5834 | 0.3402 | 21 |
| | 0.6878 | -3.1299* | | |
| **mer** | -0.3765 | -0.2308 | 0.0573 | 21 |
| | -0.3917 | -1.0745 | | |
| **mob** | -0.0587 | 0.2514 | 0.0905 | 8 |
| | -1.6168 | 0.7726 | | |
| **pep** | 0.0035 | -0.2031 | 0.0413 | 21 |
| | 0.0022 | -0.9046 | | |
| **s** | -0.2742 | -0.3202 | 0.1025 | 14 |
| | -0.3799 | -1.1706 | | |
| **txn** | 0.0259 | -0.6219 | 0.3893 | 21 |
| | 0.1985 | -3.4804* | | |
| **txu** | 0.0416 | -0.6067 | 0.3740 | 21 |
| | 0.4789 | -3.3691* | | |
| **wmt** | 0.0261 | -0.4829 | 0.2418 | 21 |
| | 0.0240 | -2.4616* | | |
| **xrx** | 0.0308 | -0.1826 | 0.0333 | 21 |
| | 0.2359 | -0.8095 | | |

**Table 9: A Test for a Constant Recovery Rate**

# Conclusion

First, equity prices can be used to infer a firm's default intensities.

Second, due to the noise present in equity prices, the point estimates of the default intensities that are obtained are not ve precise.

- This conclusion casts doubts upon the reliability of the default probability estimates obtained from structural models as in Delianedis and Geske (1998) (confirming the previous conclusion of Jarrow and van Deventer (1998, 1999) in this regard).

Third, equity prices appear to contain a bubble component, as proxied by the firm's P/E ratio. Bubbles in equity returns are no completely captured by the four factor model of Fama and French (1993, 1996).

Fourth, we compare the default probabilities obtained from equity with those obtained implicitly from debt prices using the reduced form model contained in Janosi, Jarrow, Yildirim (2000). We find that due to the large standard errors of the equity price estimates, one cannot reject the hypothesis that these default intensities are equivalent.