Measurement of Aggregate Risk with Copulas

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In measuring the risk of a portfolio, various approaches have been discussed. Models must cope with the following facts. Heteroscedasticity has been shown to be a significant determinant in modeling the log-returns of stock prices. It also has been argued, that the tail behavior of the log-return distribution can not be neglected. When aggregating the risk on a portfolio level, the specification of the dependence structure plays an important role. Here, case studies of market crashes suggest the application of concepts allowing for extremal dependence.

In this paper, we propose a multivariate model for describing a set of risk factors that determine a portfolio. We follow a copula approach, i.e. the modeling is done in two steps. First, the marginal distribution of each risk factor is specified, where we apply a GARCH-type model to capture conditional heteroscedasticity effects. The innovations of this time-series are modeled by a parametric distribution function allowing for different types of tail behavior also including heavy tails. In a second step, the joint distribution of all risk factors is established by a copula function that operates on the innovations of the time-series of each risk factor. The copula function used in our approach is the Frank copula with a transformed generator. The transformation of the generator of the copula enables us to parametrize the dependence structure in a flexible way. In particular extremal dependence can be covered.

Finally, we apply the theoretical framework presented in this paper to financial data, primarily using log-returns of German stocks as risk factors. The parameters of a bivariate model are estimated and the dependence structure is analyzed. By performing a goodness-of-fit test, the quality of the estimation is examined. The results are compared to standard approaches based on the Variance-Covariance concept. Furthermore, the impact of the proposed model on practically relevant risk measures is studied.

JEL Classification: C22, C51, C52, C53.

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