ELECTRICITY DERIVATIVES

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Some modeling choices

- Prices are constant within each unit of time (hour in N.A., half day in Germany). Therefore discrete models seem to be preferable to diffusion models.
- Mean reversion is extreme after major price jumps, especially in N.A. Therefore it may be useful to model extreme events differently.

A spot process for electricity prices

- **Electricity prices p for each period)t follow a diffusion with ‘spikes’:**

  dx=m)t+v)z

  p=f(x)+wj

- **w equals 0 with probability (1-q), otherwise is 1. The spike j is normally distributed with mean>> 0.**
- **The price process is non-Markovian. Its future evolution depends on its past after a spike. It may be described as Markovian in 3 variables (p,w,j). However we treat it as non-Markovian heuristically.**

Features of the spot process

- The main feature of our price process is that spikes, due to outages, disappear without permanent effects at the end of the period they occur.
- Spikes are positive. The probability of a spike following immediately after an other one is small.
- Seasonalities are incorporated in the diffusion.

Pricing forward contracts

- A forward contract on electricity delivers one unit of power over the last time period.
- The forward price is the expected price under risk neutrality:
- $f_T = E(f(x_T) + w_T j_T)$

Pricing Swaps

- A swap is a series of forward contracts. The value of a swap is
- swap=PVG((f_T)-X)
  where X is the current swap price for that maturity. X is determined from current forward prices by solving swap=0.

Pricing European Call Options

- An European call option is influenced by a spike at the last period only. Its value is:
- PV((max(p_T-X),0)
  where $p_T = f(x_T) + w_T j$
- The above values may be computed by simulating spikes on binomial trees.

Pricing European Put Options

- The value of European Put Options is influenced by spikes in the last period only.
- The European put value is:
- $PV(E(max(X-p_T),0))$
- values are computable by simulating spikes on binomial trees.

The American Call Option Problem

- The early exercise of the American call may be determined by simulation. That is cumbersome because a new simulation must begin from each node.
- A faster approximation starts from the binomial tree (u,d,B)without dividends. The following conditions apply:
- $f'm+E(w)E(j)<r$ (necessary)
- $X \leq pd^{(T-t)}$ (sufficient)
- $c=p-X$ (necessary and sufficient)

The Am. Call Approximation

- Our approximation exercises the option if $X < pd^{(T-t)}$.
- Note that at t=T the European condition applies
- A comparison with the optimal policy finds that at short maturities the approximation is adequate.
- The approx. value of the American Call is found by simulating spikes on the binomial tree and applying the early exercise condition

The American Put Option Problem

- The early exercise of the American put is enhanced by the possibility of spikes in future.
- The conditions for early exercise are:

- $X \geq pu^{(T-t)}$ (sufficient)
- put=X-p (necessary and sufficient)

The Am. Put approximation

- Our approximation exercises the option if
- $X \geq pu^{(T-t)}$ (sufficient condition)

- Values can be found by simulating spikes on a binomial tree.

The perfect foresight value

- An upper boundary to value, that allows for a conservative evaluation of approximations, is the perfect foresight value. It computes for each simulation path the supremum of the derivative pay-off. That is much faster than running an auxiliary simulation from each node.

Conclusion

- Preliminary evidence suggests that our approximations are very good in the presence of the extreme spikes common in North America. Their suitability for less volatile European markets is left to future research.