Macquarie University Applied Finance Centre
frank.ashe@mafc.mq.edu.au

History

It has been known for many years that the most common quantitative method for portfolio construction – a mean-variance optimiser, maximising $U = r - \frac{1}{2}\lambda\sigma^2$ – is very sensitive to the value of the inputs[1]. This led to a general distrust of quantitative models by many non-quantitative practitioners, and attempts by the quantitatively inclined practitioners to overcome the readily perceived problems:

- constraining solutions to lie within "reasonable" ranges – what's reasonable?;
- moving to assessing returns against a benchmark – how do we choose the benchmark?;
- resampling historical data[2] - is historical data an accurate reflection of the future?;
- shrinkage estimators such as implied by the Bayesian interpretation of Black-Litterman[3], or directly in Chopra, Hensel and Turner[4] - is the point to which we shrink the most suitable?

Many of the attempts in the 1970s and 1980s to overcome the problem ignored the most basic of reasons as to why the optimiser failed: its basic assumptions were violated. There has been a lot of discussion about some of the basic assumptions: are we only interested in the first two moments of future wealth distribution; how well does the Normal or lognormal distribution describe the distribution of future wealth (therefore obviating the question of interest in moments); does volatility effectively measure risk; how can we assess lambda. What generally was ignored was the most fundamental assumption of all, that we know the values of the underlying parameters over which we are optimising – we don't.

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1 For example, see the oft quoted Chopra, V. K. and W. T. Ziemba (1993) *The Effects of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice* Journal of Portfolio Management 6-11. This paper concentrates on the position of a diversified stock portfolio, but the effect is more general. The effect is less noticeable, but still economically significant, for a simple equity versus bond portfolio.

2 See Jorion, P. (1992) *Portfolio Optimization in Practice* Financial Analysts Journal January-February 68-74 for an early reference.

3 Black, F. and R. Litterman Ibid.*Global Portfolio Optimization* September-October 28-43

4 Chopra, V. K., C. R. Hensel and A. L. Turner (1993) *Massaging Mean-variance Inputs: Returns from Alternative Global Investment Strategies in the 1980s* Management Science **39** 7 845-855

How profound is our ignorance?

Historical data

Just how unsure are we of the basic parameters of our investment asset classes. We are in a much better position to answer that question after the turn of the millennium prompted a number of researchers to examine the 20th century from a financial viewpoint:

- Jorion and Goetzmann[5] (JG) review the history of developed markets in twentieth century and show how unusual the history of the US has been. Using a broader set of markets we may be justified in lowering the risk premium from what we observe in the highest ranked market.
- Dimson, Marsh and Staunton[6] (DMS) also review the history of the developed markets in the twentieth century and come up with a more sanguine view than Jorion and Goetzmann.

The following table shows their respective estimates for the real equity return above consumer price inflation.

[5] Jorion, P. and W. N. Goetzmann (1999) *Global Stock Markets in the Twentieth Century* Journal of Finance **54** 3 953-980

[6] Dimson, E., P. Marsh and M. Staunton (2002) *Triumph of the Optimists: 101 Years of Global Investment Returns* Princeton University Press

Comparative Estimates of Returns

Country	Real Price Return	Real Total Return	Volatility	Volatility
	Jorion and Goetzmann	Dimson et al	Jorion and Goetzmann	Dimson et al
Australia	1.58	7.5	13.94	17.7
Belgium	-0.26	2.5	18.97	22.8
Canada	3.19	6.4	16.65	16.8
Denmark	1.87	4.6	12.69	20.1
France	0.75	3.8	21.25	23.1
Germany	1.91	3.6	23.53	32.3
Ireland	1.46	4.8	15.02	22.2
Italy	0.15	2.7	25.66	29.4
Japan	-0.81	4.5	17.77	30.3
Netherlands	1.55	5.8	14.80	21.0
South Africa	-1.76	6.8	15.89	22.8
Spain	-1.82	3.6	16.00	22.0
Sweden	4.29	7.6	16.65	22.8
Switzerland	3.24	5	14.73	20.4
United Kingdom	2.35	5.8	15.68	20.0
United States	4.32	6.7	15.84	20.2

Average	1.38	5.11	17.19	22.74
Std Dev	1.85	1.57	3.42	4.21
Median	1.57	4.90	15.95	22.10

Jorion & Goetzmann's results are for real price returns and are mostly for periods from 1921 to 1996. The shortest period is 1947-96 for South Africa. Dimson et al have total real returns for the period 1900 to 2000. Both returns are geometric averages.

Standard deviations for JG are based on monthly return data annualised in the usual manner. Where they reported two separate periods for Germany and Japan I've taken a weighted average of the variances using the length of the period as the weight. DMS volatility are based on annual returns so any positive autocorrelation in the monthly data will lead to a larger standard deviation for returns than for JG. The additional return from dividends should not cause a large change to the volatility.

DMS data is biased(?) by the exclusion of data from Latin America, Egypt, India, Pakistan, and Eastern Europe. This gives DMS a much higher average for their sample than JG's. More worrying is the difference in the volatilities. Without being

able to delve into the data it's hard to tell whether the large difference between the studies comes from a positive monthly autocorrelation or the different data periods.

How do we use this in practice? This is the sort of data that we need to construct portfolios for the long term, and here I'm thinking primarily of portfolios for retirement. Even if we are going to be continually monitoring and changing the portfolios depending on changing circumstances, we need to start off on the right foot - especially concerning a reasonable savings rate. Using shorter periods of data will more likely lead us to myopic asset allocations that are too driven by what has happened in the recent past and not what could happen in the future.

Basic Statistics

We want to look into the future, so how good is our lens on the past? If we calculate an estimate of, say, expected real return for equity markets based on an historical observation of 100 years of data then we must remember that our past estimate is subject to sampling error – on the basis that we assume that there is the same random process that delivers returns in the future and the past and we see only one sample of past data from which to estimate the parameters of the process.

The standard error of the mean return is large, viz the standard deviation of the process divided by the square root of the number of observations; for most countries in the DMS database this gives around 2% standard error for the real equity return over the last 100 years. Interestingly, this value is around the level of the standard deviation of the long-term returns across countries given at the bottom of the previous table.

How significant is this standard error? Using a simple bond versus equity allocation example, a 2% increase in expected equity return can change the allocation to equities by nearly 20%!

What values could we use?

Let's try to estimate values to use for a substantial fraction of the 21st century, as we would have to do if advising a 20 year old on their superannuation contributions. Because the DMS data includes dividends we will use it in preference to the JG data. We will make a few broad assumptions at the end to become more pessimistic.

In the discussion below the expected value of the real return for equities is adjusted from the geometric return above by adding half the variance. We cannot guarantee that our countries in which we invest will not, in the 21st century, go through similar strength upheavals to what we have seen in the 20th and so estimates of the mean return and volatility are interlinked.

The numbers are suitable for inputs for a portfolio invested in individual countries. A multiple country asset class would have lower volatility because of the diversification effect. Standard errors for the expected values are included as these should be inputs to any serious optimisation undertaken. Remember that over long periods, estimation errors can have a larger contribution to uncertainties in estimates of final wealth than the underlying market volatility.

In assessing inputs we should also remember that we want estimates of the future values and not estimates of the past – we look at the past hoping it will give some idea as to what could occur. If we see some event in the past that we believe will not reoccur then we should adjust estimates based on the past to remove that effect.

One such effect has been the reduction in dividend yields across many countries. This has had the effect of increasing the PE ratio of equities at the end of the period to the PE at the beginning. If we believe the reduction will not occur in the future we should reduce the expected returns by the portion of the observed historical price returns that can be explained by this phenomenon. We may also have seen a reduction in the risk premium required for equities. Neither of those two effects has been incorporated in the examples below. To do so could reduce the expected returns[7] by around 2%.

Optimistic scenario

The future stock markets will behave like those of the more fortunate countries over the last century, volatility is now at DMS level for that period. The expected real return has increased because I've assumed the geometric mean doesn't change, so the expected return increases in line with the variance squared.

Expected values	Real Return	8.5%
	Volatility	20%
Std Errors	Real return	1.0%
	Volatility	2%

21st century = 20th rerun

The current century will be a rerun of the last century. Some countries will be blighted in some manner. We use averages from DMS's full table.

Expected values	Real Return	7.75%
	Volatility	23%
Std Errors	Real return	1.75%
	Volatility	4%

Pessimistic scenario

There is the possibility that the market of interest will suffer major disruption for significant periods of time. Expected returns are more suggestive of JG data.

Expected values	Real Return	7.0%
	Volatility	25%
Std Errors	Real return	3%
	Volatility	5%

The standard error of the volatility is most probably understated as volatilities of much more than 35% have been observed for some markets.

Most optimistic scenario

The future stock markets will behave like those of the more fortunate countries over the last three-quarters of last century, with returns and volatility as suggested by JG. A 4% dividend has been added to JG data, this being around the average for MSCI data for these countries since 1970.

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[7] See Arnott, R. D. and P. L. Bernstein (2002) *What Risk Premium is "Normal"* Financial Analysts Journal , or Chen, P., G. T. Baierl and P. D. Kaplan (2002) *Venture Capital and its Role in Strategic Asset Allocation* Journal of Portfolio Management **28** 2 83-89 for the arguments.

Expected values	Real Return	8.25%
	Volatility	16%
Std Errors	Real return	1.0%
	Volatility	1%

Why does this "Most optimistic" scenario have a lower return than the "Optimistic" scenario? My optimism has to do with the volatility of the returns, which has dropped from 20% to 16%. This reduction in volatility means that (assuming a Lognormal distribution) the median return has increased from the "optimistic" to the "most optimistic" portfolios. I have used the average of the geometric returns of the countries and derive the arithmetic average from that using the volatility.

Discussion

There is a high degree of uncertainty associated with the most basic parameters of risk and return in the investment world. A reasonable confidence interval for the expected real return on the stock market must be at least 3% wide, possibly much wider. Our uncertainty in future standard deviations is just as profound.

What is the range of "optimum" asset allocations that satisfy these values? Assume a range of 6% to 9% for the future expected real return for equities and 16% to 25% for the volatility. Assume an expected 3% real return for bonds in every scenario and 6% volatility, and a correlation in real return of 0.4 between the two assets. The risk aversion coefficient is 4 and the expected return minus variance times half the risk aversion is maximised. The following table shows the percentage invested in equities for this range of possible inputs.

	Equity volatility		
Real equity return	16%	20.5%	25%
6%	33.7%	17.3%	9.6%
7.5%	51.2%	27.8%	16.4%
9%	68.6%	38.2%	23.3%

We cannot pretend to know more parameters much more precisely than this. We are left with the conclusion that we cannot know such a basic parameter of our investment portfolio as the percentage in equity assets to within more than 15%.

Alternative Asset classes

The preceding discussion covers the most examined asset class in history, across many countries, with abundant data that covers a range of economic scenarios. Yet we cannot come up with a precise estimate of how much of that asset should be within a portfolio. What chance do we have to come up with a precise estimate when we deal with "alternative" asset classes for which we have much less data?

For example, how much of a diversified portfolio should be allocated to hedge funds? We are lucky to have ten years of data for an individual fund; we cannot use index data to model the portfolio behaviour of an individual fund because idiosyncratic risk is so high; we need to be very worried about survivorship bias[8]; the return distribution

[8] See Barry, R. (2003) *A Walk Through the Hedge Fund Graveyard* Macquarie University Applied Finance Centre MAFC Paper

is distinctly asymmetric and fat tailed; we can have no pretence of optimising in such a situation, satisficing is the best that we can manage.

Private equity data is even more difficult to come by[9].

How to acknowledge ignorance

Words

How can we take this ignorance of basic parameters into account when we try to ascertain a portfolio in which to invest, either for a short horizon or a long horizon? To be completely honest with ourselves as an industry we must adopt a position of humility; stop using language that implies the portfolios we adopt are optimum in anything but some technical, and impractical, sense; stress far more often than we do that our recommendations on portfolio structure are more like vague suggestions than the best bet that can be made at the time; and make our customers realise the implications of the expanding funnel of doubt covering the portfolio returns that will be basis of their retirement.

In addition, as we know that people tend to be overconfident in their abilities, by acting as a countervailing tendency the industry should end giving their clients – remember those people? – a portfolio better suited to their future.

The next section discusses some of the approaches to portfolio construction that take the previous section detailing our ignorance.

Deeds

Two basic questions that first need to be answered are, for what horizon are we investing, and what assets can we invest in. I'll take a long term perspective – life cycle investing is by far the most important decision people will face and single term investment questions can be considered a subset of this more difficult problem. For the second question I'll assume that future earning power is included as one of the major assets of a young person, which depreciates at a steady rate; lifestyle asset choices, such as buying a home, will be ignored.

Changing asset allocations

Including future income as an asset leads to a changing asset allocation as a person ages[10]. The future income acts in an analogous fashion to a fixed income instrument and so when we look at the full portfolio of an investor, taking this depreciating asset into consideration, the balance between bonds and equities is on a more even keel over a persons life, notwithstanding that the proportionate allocation financial assets may change radically.

This actually got ahead of ourselves here – what inputs were used to come up with the changing asset allocations? If we assume the same degree of uncertainty in the parameters as in the earlier example then there is a similar degree of uncertainty in the allocation between the two major asset classes. Given this similarity in qualitative

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[9] See Kaplan, S. and A. Schoar (2003) *Private Equity Performance: Returns, Persistence and Capital* NBER W9807

[10] See Campbell, J. Y. and L. M. Viceira (2002) *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors* Oxford University Press for a recent discussion of this problem.

results, the rest of this section will discuss only a single horizon portfolio construction.

Methodologies

A number of approaches have been undertaken to tackle our ignorance:

- **Disregard the problem, pretend we do know what we are talking about** - This is by far the most common approach and is in line with the observation in the behavioural finance literature that we are congenitally prone to overconfidence. And as those overconfident people who can project confidence are usually perceived to be better managers[11] than people with low self-confidence, then money will tend to be invested by the public with managers who don't acknowledge that they are ignorant[12].

  In most cases I'd say the disregard is not intentional; the depth of ignorance doesn't even cross the mind.

- **Equilibrium approaches**[13] - If we don't know what's happening then we can use the combined wisdom of all the market's participants. For clearing securities markets, to a first approximation, the capitalisation of any market segment takes into account the expected returns, risk, and non-marketable assets of all unconstrained market participants.

  Leveraging this market portfolio up or down allows a manager to set their risk-return tradeoff to their desired level.

  If we can identify that we are the member of a specific subset of market investors, say Australian government bond investors or US Domestic asset investors, then the previous argument can be extended. We can assume that our peer group, by allowing the market to clear where it has, has incorporated their specific knowledge of this peer group's risk and reward characteristics into their asset and security allocation. Adoption of this allocation acknowledges our ignorance and seeks to maximise return by reducing investment costs; indexation is one approach that fits a naïve adoption.

  If we believe we have specific knowledge not incorporated in market prices on any particular security or market sector then this view can be incorporated into our portfolio by tilting our portfolio. This tilting can take into account the appropriate uncertainty in our own views. Black and Litterman's technique[14] is one such mechanism. The process of incorporating our view usually requires some opinion on the second moments of the distribution of future returns – we don't usually acknowledge complete ignorance.

  But in one sense this second moment information is of second-order importance. Using the Black-Litterman process as an example, the assumed covariance

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[11] In fact, they're usually judged to be better at most things.

[12] One caricature of behaviour seen in the recent tech bubble is that many managers who proclaimed that they didn't know what the broad market was doing, apart from being priced at ridiculous levels, suffered outflows of money. It flowed to those people who were supremely confident in the new economy.

[13] A recent compendium of one group's approach is given in Litterman, R. (2003) *Modern Investment Management: An Equilibrium Approach* Wiley

[14] Black, F. and R. Litterman (1992) *Global Portfolio Optimization* Financial Analysts Journal September-October 28-43

matrix is used in running the mean-variance algorithm backwards from the asset allocation to implied returns; these implied returns are combined with one's views on the correct levels of these returns and put back through the mean-variance optimiser. The view on the covariance matrix only has an effect on the tilt, not on the basic asset allocation.

- **Shrinkage methods** – Some methodologies are based on the premise that the market capitalisation is not a correct incorporation of the information in the market place, or that some mechanical approach to estimating the covariance matrix gives a better estimate for inputting into some optimisation process.

  One approach, taken by Chopra, Hensel and Turner[15], takes a Stein estimator approach and shrinks estimates of the mean returns and correlations from their historical averages[16] towards grand historical averages. One rationale for this is their acknowledgement of the large statistical uncertainty pertaining to historical estimates of the average returns.

  We don't have to shrink only historical averages towards some grand mean; Stein estimator-type effects occur in the context of other estimators of means and covariances[17] and so shrinking estimates of means, variances and correlations towards grand means is likely to give better estimates for input into a mechanical optimiser.

  Arnott[18] takes a different approach. If we take as a working hypothesis that prices may not be an accurate reflection of "true" value then market capitalisation will not truly reflect how much of the asset should be in a portfolio[19]. He proposes, for a normal[20] portfolio, shrinking market capitalisation towards other estimates of company size – gross dividends paid, book value, income, sales, and number of employees. Overvalued companies will tend to form a smaller part of the portfolio than they would for capitalisation weighting and undervalued companies will form a greater part. Studies on the US market indicate that over the long-term this approach to constructing a normal portfolio adds approximately 2%-3% p.a. compared to broad market based indexed equity portfolios.

- **Statistical** – In this grab-bag I would include methods such as treating optimised portfolios as a statistic of the distribution of the parameters assumed

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15 Chopra, V. K., C. R. Hensel and A. L. Turner (1993) *Massaging Mean-variance Inputs: Returns from Alternative Global Investment Strategies in the 1980s* Management Science **39** 7 845-855

16 Assuming, for the purpose of argument, that the recent historical performance is a reasonable estimate of the future performance.

17 The basic argument runs along the lines that in a series of estimates of parameters the extreme large and small values will typically have the largest errors, over and under respectively, and shrinking towards a grand average will reduce the estimation errors in aggregate, though not necessarily for any one particular estimate. Sufficiently, but not unreasonably, low correlation of estimators is required.

18 Arnott, R. D., J. Hsu and P. Moore (2004) *Fundamental Indexation* Research Affiliates http://www.researchaffiliates.com/articles/cms/indexation.pdf

19 CAPM and other models usually assume no uncertainty in estimates of their parameters.

20 Used in the sense of a portfolio that one reverts to in the absence of any particular view on the merits of any asset. Arnott et al proposes using this normal portfolio as an "indexed" portfolio, but there is nothing stopping the portfolio being adopted as the base from which a traditional active management would place their relative bets.

(usually historical in some form, even if only in the distribution of residuals) and bootstrapping off the parameter distributions to gain a distribution of "optimal" portfolios.

- **Robustness** – Let's throw out all concepts of looking for the best return in the mathematical or formulaic optimisation of our portfolio and concentrate on the reduction of risk – however defined. Maximisation of return comes through the restriction of the asset set, suitably loosely defined, to those assets expected to give good performance, again loosely defined,; or by checking that a particular specified portfolio, which presumably has good upside potential or no-worse potential than the naïve alternative, has suitable risk characteristics. Many of the current benchmark-agnostic, long-only equity managers and hedge fund managers would fit within this loose definition.

  One of the attractions of this approach is that it removes the estimation error of the mean return from the reach of the optimiser, which in this context tends to be an estimation error maximiser.

So, what observations can we make concerning the future of investment management?

Predictions

I'll chance my arm on the following predictions:

- The industry trend will be towards greater acknowledgement of the inherent uncertainty. The movement will be slow and reluctant; one of the main drivers will be activity by regulators to ensure that the public are properly informed when making investment decisions.
- More emphasis will be made that the industry constructs portfolios that have specific risk characteristics and that are believed to have desirable upside characteristics. We'll move from casting ourselves as maximisers to calling ourselves satisficers, using the terminology of Schwartz[21].
- This latter movement will use more quantitative resources than have been used in the past.
- Indexing as currently undertaken will diminish in importance, replaced by management around normal portfolios based on the likelihood that capitalisation weighted benchmarks are not necessarily the most efficient form of investing. There is no good term for what will evolve – it's not *passive*, it's not *indexed*, it's not *active*, but it's the future.

And that is a good note on which to end.

[21] Schwartz, B. (2004) *The Paradox of Choice: Why Less is More* HarperCollins

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